MOCK EXAM 10

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

Name: _____

(2 1/4 hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your name in the space provided on Page 1.

2. This paper consists of **THREE** sections, A(1), A(2), and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your name on the graph paper and supplementary answer sheets.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

SECTION A(1) (35 marks)

1. Simplify $(\alpha^2\beta)(\alpha^3\beta^{-2})^{-4}$ and express your answer with positive indices. (3 marks)

2. Make b the subject of the formula $\frac{2}{a} - \frac{3}{b} = 4$. (3 marks)

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3. Factorize

(a) $x^2 + xy - 20y^2$,

(b) $x^2 + xy - 20y^2 - 8x + 32y$ (3 marks)

4. (a) Find the range of values of x which satisfy both $\frac{5-x}{3} > 2x + 1$ and $3x + 5 \geq 0$.

(b) Write down the greatest integer satisfying both inequalities in (a). (4 marks)

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5. The number of marbles owned by a girl is 4 times that owned by a boy. If the girl gives 22 of her marbles to the boy, then the number of marbles owned by the boy is 3 times that owned by the girl. Find the total number of marbles owned by the boy and the girl. (4 marks)

6. The marked price of a ring is $780. It is given that the marked price of the ring is 30% higher than the cost.

   (a) Find the cost of the ring.

   (b) If the ring is sold at $720, find the percentage profit. (4 marks)

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7. In a polar coordinate system, O is the pole. The polar coordinates of the points A and B are $(r, 75^o)$ and $(r, 165^o)$ respectively, where r is a positive constant. It is given that the distance between A and B is $6\sqrt{2}$. Find

(a) $\angle AOB$,

(b) r,

(c) the area of ΔOAB. (4 marks)

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8. In Figure 1, *ABE* and *CDE* are straight lines. It is given that $\angle ACE = \angle DBE$.

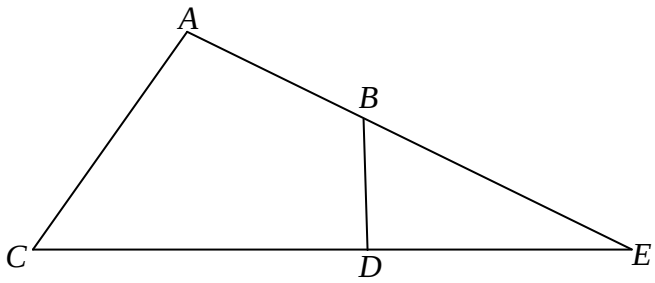

Figure 1

(a) Prove that $\Delta ACE \sim \Delta DBE$.

(b) It is given that $AC = 40$ cm, $AE = 75$ cm, $CE = 85$ cm and $DE = 45$ cm.

(i) Is ΔACE a right-angled triangle? Explain your answer.

(ii) Find the area of the quadrilateral *ACDB*. (5 marks)

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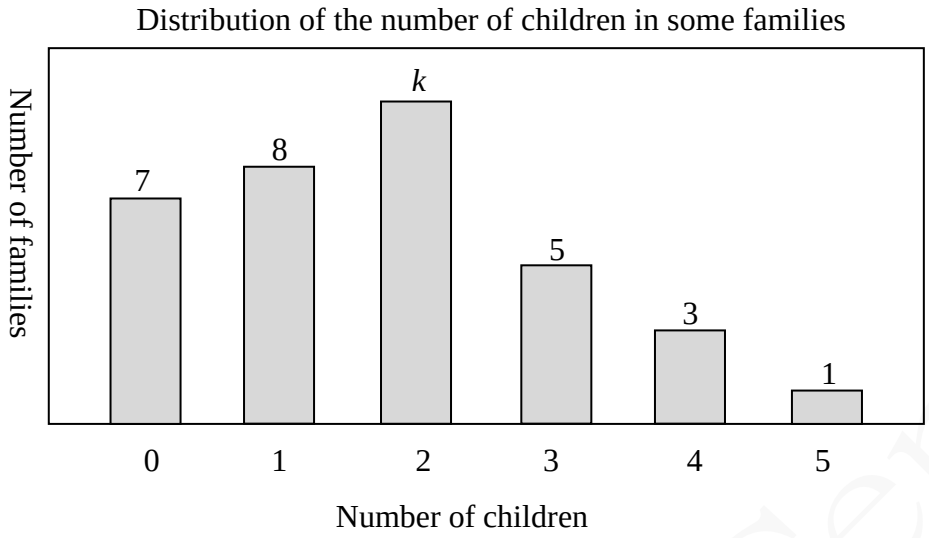
9. The bar chart below shows the distribution of the number of children in some families of a building.



If a family is randomly selected from the families, then the probability that the selected family has less than 3 children is $\frac{3}{4}$.

(a) Find k.

(b) Write down the median, the inter-quartile range and the standard deviation of the distribution.

(5 marks)

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SECTION A(2) (35 marks)

10. The cost of making a square handkerchief with a side of s cm is $\$C$. C is partly constant and partly varies as s^2. When $s = 15$, $C = 65$ and when $s = 20$, $C = 100$.

    (a) Find the cost of making a handkerchief with a side of 18 cm. (4 marks)

    (b) Someone claims that the total cost of making two handkerchiefs with a side of 9 cm is higher than the cost of making a handkerchief with a side of 18 cm. Is the claim correct? Explain your answer. (2 marks)

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11. The stem-and-leaf diagram below shows the distribution of the numbers of weekly hours spent by a group of students in doing homework:

Stem (tens)	Leaf (units)									
1	5	7	8	9	9	9				
2	1	2	2	3	3	4	4	4	*a*	8
3	2	*b*	*b*	*b*						

The mean of the distribution is 24.

(a) Find *a* and *b*. (3 marks)

(b) Write down the greatest possible range of the distribution. (1 mark)

(c) Find the least possible inter-quartile range of the distribution. (3 marks)

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12. Let $f(x) = 2x^3 + kx^2 + 5x + 4$, where k is a constant. It is given that $f(x) \equiv (2x + 1)(ax^2 + bx + c)$, where a, b and c are constants.

(a) Find a, b and c. (4 marks)

(b) Someone claims that all the roots of the equation $f(x) = 0$ are real numbers. Do you agree? Explain your answer. (3 marks)

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13. The coordinates of the points E, F and G are (–5, 6), (2, 9) and (7, –3) respectively. The circle C passes through E and the centre of C is G.

(a) Find the equation of C. (2 marks)

(b) Prove that F lies inside C. (2 marks)

(c) Let H be a moving point on C. When H is closest from F,

(i) describe the geometric relationship between F, G and H;

(ii) find the equation of the straight line which passes through F and H. (3 marks)

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14. The base radius of the solid right circular cylinder X and the base radius of the solid right circular cone Y are equal. The heights of X and Y are 24 cm and 48 cm respectively. The volume of the solid right circular cone Z is equal to the sum of the volume of X and the volume of Y. The base radius of Z is equal to the base diameter of X. A craftsman finds that the volume of Y is $6\,400\pi$ cm^2.

(a) Find the base radius of Y. (2 marks)

(b) Are Y and Z similar? Explain your answer. (3 marks)

(b) The craftsman claims that the sum of the curved surface area of X and the curved surface area of Y is equal to the curved surface area of Z. Do you agree? Explain your answer. (3 marks)

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SECTION B (35 marks)

15. A queue is randomly formed by 6 boys and 4 girls.

(a) How many different queues can be formed? (1 mark)

(b) Find the probability that no girls are next to each other in the queue. (3 marks)

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16. The straight lines L_1 and L_2 are perpendicular to each other. The y-intercept of L_1 is 5. It is given that L_1 and L_2 intersect at the point (12, –4). Let R be the region (including the boundary) bounded by L_1, L_2 and the y-axis.

(a) It is given that R represents the solution of a system of inequalities. Find the system of inequalities. (3 marks)

(b) Find the largest value of $4x - 6y$, where (x, y) is a point lying in R. (2 marks)

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17. The general term of an arithmetic sequence is denoted by A(n) where n is a positive integer. It is given that A(3) = 22 and A(10) = 50.

(a) Find A(1). (2 marks)

(b) Suppose that $\log_3 B(n) = A(n)$ for any positive integer n.
Find the greatest value of k such that $\log_{27}[B(1)B(2)... B(k)] < 2\,023$. (5 marks)

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18. (a) A thin metal sheet *ABCD* is in the shape of a quadrilateral. It is given that $AB = 50$ cm, $BC = CD$, $\angle BAD = 40^\circ$, $\angle ABC = 130^\circ$, $\angle BCD = 100^\circ$ and $\angle ADC = 90^\circ$. Find *CD*.

(2 marks)

(b) The metal sheet *ABCD* described in (a) is now given. Let *E* be a point lying on *AD* such that *BE* is perpendicular to *AD*. The metal sheet is folded along *BE* such that *AE* is perpendicular to the plane *BCDE*. Three thin triangular metal sheets are placed to this folded metal sheet to form a pyramid (see Figure 2).

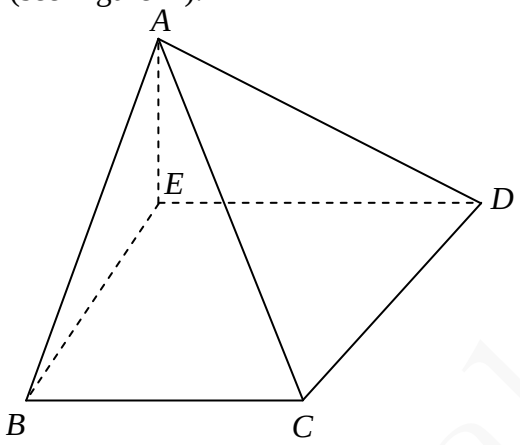

Figure 2

(i) Find $\angle BAC$.

(ii) Does the angle between the plane *ABC* and the plane *BCDE* exceed 50°? Explain your answer.

(5 marks)

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19. The coordinates of the centre of the circle C are (16, 25). Denote the radius of C by r. Let L be the straight line $12x + ky - 148 = 0$, where k is a constant. It is given that L is a tangent to C.

(a) Find the equation of C in terms of r. Hence, express r^2 in terms of k. (4 marks)

(b) L passes through the point $D(-11, 56)$.

(i) Find r.

(ii) It is given that L cuts the x-axis at the point E. Let F be a point such that C is the inscribed circle of $\triangle DEF$. Is $\triangle DEF$ an obtuse-angled triangle? Explain your answer. (8 marks)

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END OF PAPER

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