MOCK EXAM 2
MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

Name: _____

$(2\frac{1}{4}$ hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your name in the space provided on Page 1.

2. This paper consists of **THREE** sections, A(1), A(2), and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your name on the graph paper and supplementary answer sheets.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

SECTION A(1) (35 marks)

1. Simplify $\dfrac{(x^{-2}y)^{-2}}{x^3 y^2}$ and express your answer with positive indices. (3 marks)

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2. Make m the subject of the formula $\dfrac{2}{m} + \dfrac{1}{n} = 3$. (3 marks)

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3. Factorize

   (a) $m^2 - 4m - 5$,

   (b) $mn + n + m^2 - 4m - 5$. (3 marks)

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4. (a) Round up 759.2489 to the nearest hundred.

   (b) Round down 759.2489 to 3 significant figures.

   (c) Round off 759.2489 to 3 decimal places. (3 marks)

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5. Consider the compound inequality

$$2 - x < \frac{5-3x}{2} \text{ or } 5 + x > 6 \quad \ldots\ldots (*).$$

(a) Solve (*).

(b) Write down the smallest positive integer satisfying (*). (4 marks)

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6. The selling price of a bag is \$256. The bag is sold at a discount of 20% on its marked price and the percentage profit is 60%.

(a) Find the marked price of the bag.

(b) Find the profit of the bag. (4 marks)

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Answers written in the margins will not be marked.

7. In Figure 1, *BD* is a diameter of the circle *ABCD*. If $AB = AC$ and $\angle ABD = 22^{\circ}$, find $\angle AEB$.
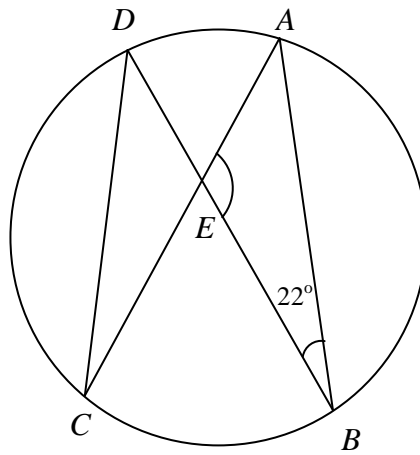
Figure 1

(5 marks)

Answers written in the margins will not be marked.

8. The coordinates of the points A and B are $(-3, 8)$ and $(6, -5)$ respectively. A' is the reflection image of A about the x-axis. B is rotated clockwise about the origin O through 90° to B'.

   (a) Write down the coordinates of A' and B'.

   (b) Let P be a moving point in the rectangular coordinate plane such that P is equidistant from A' and B'. Find the equation of the locus of P. (5 marks)

9. The following table shows the distribution of the number of children in some families.

Number of children	0	1	2	3	4
Number of families	5	7	a	7	b

It is given that a and b are positive numbers.

(a) Find the least and greatest possible values of the inter-quartile range of the distribution if there are no more than 50 families.

(b) If $b = 8$ and the median of the distribution is 3, find all possible values of a. (5 marks)

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SECTION A(2) (35 marks)

10. City *X* and city *Y* are 180 km apart. Figure 2 shows the graphs for car *A* and car *B* travelling on the same highway between city *X* and city *Y* during the period 8:30 to 11:30 in a morning. Car *A* travels at a constant speed during the period. Car *B* comes to rest at 9:45 in the morning.
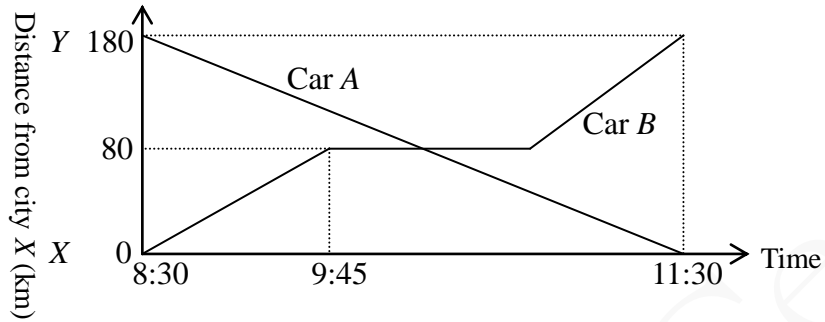

Figure 2

(a) Find the distance of car *A* from city *Y* at 9:45 in the morning. (2 marks)

(b) At what time in the morning do car *A* and car *B* meet? (2 marks)

(c) The driver of car *B* claims that the average speed of car *B* is higher than that of car *A* during the period 9:45 to 11:30 in the morning. Do you agree? Explain your answer. (2 marks)

11. A survey is conducted on the number of hours spent on watching TV in a week by a group of students. The stem-and-leaf diagram below shows the distribution.

Stem (tens)	Leaf (units)
0	8 9 9
1	0 0 2 2 3 3 3 4 5 6 7 8 8
2	0 1 1 2 4

(a) Find the mean and median of the above distribution. (2 marks)

(b) Three students join the survey. It is found that the mean of the number of hours by these three students is 15. It is found that the number of hours by one of them is 17.

   (i) Write down the mean of the number of hours by all students.

   (ii) Is it possible that the median of the number of hours is the same as that found in (a)? Explain your answer. (4 marks)

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12. (a) Find the value of k such that $x - 1$ is a factor of $kx^3 - 18x^2 + 28x - 13$. (1 mark)

(b) Figure 3 shows the graph of $y = -3x^2 + 12x - 4$. Q is a variable point on the graph in the first quadrant. P is the foot of the perpendicular from Q to the x-axis and R is a point on the graph such that QR is parallel to the x-axis.
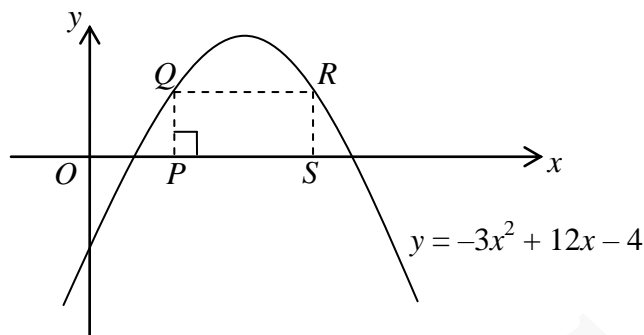

Figure 3

(i) Let $(m, 0)$ be the coordinates of P. Express the area of the rectangle $PQRS$ in terms of m.

(ii) Are there three different positions of Q such that the area of the rectangle $PQRS$ is 10? Explain your answer. (6 marks)

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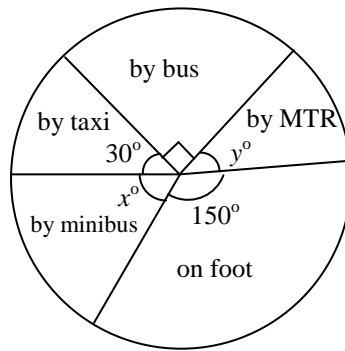
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13. The following pie chart shows the distribution of transportation taken by a group of students to school.

Distribution of transportation taken by a group of students to school

If a student is randomly selected from the group, the probability that he/she goes to school by MTR is $\dfrac{7}{60}$.

(a) Find the values of x and y. (2 marks)

(b) There are 125 students going to school on foot. How many students go to school by MTR? (2 marks)

(c) If 60 more students join the group, can the angle of the sector representing "by MTR" be doubled? Explain your answer. (3 marks)

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14. The equation of the circle C is $x^2 + y^2 - 8x - 6y - 96 = 0$. Denote the centre of C by D.

   (a) Write down the coordinates of D. (1 mark)

   (b) The equation of the straight line L is $4x + 3y - 75 = 0$. It is found that L intersects C at two points A and B. Denote the mid-point of A and B by M.

       (i) Find the distance between D and M.

       (ii) O is the origin.

           (1) Describe the geometric relationship between O, D and M. Explain your answer.

           (2) Find the ratio of the area of $\Delta\, ODA$ to the area of $\Delta\, DMA$. (8 marks)

Answers written in the margins will not be marked.

SECTION B (35 marks)

15. The scores of Mary and John in a test are 68 marks and 48 marks respectively. Their standard scores are 2 and –3.

    (a) Find the standard score of Peter who gets 80 marks in the same test. (2 marks)

    (b) Zoe who gets the highest mark is found to have cheated in the test and is disqualified. Her mark is then deleted. Will there be any change in their standard scores due to the deletion of the test score? Explain your answer. (2 marks)

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16. In Figure 4, the 1st pattern consists of 2 dots. For any positive integer n, the $(n + 1)$th pattern is formed by adding 3 dots to the nth pattern. Find the least value of m such that the total number of dots in the first m patterns exceeds 14 455. (4 marks)
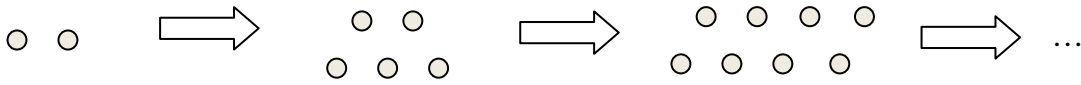

Figure 4

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17. Let $f(x) = -x^2 + 4kx - 5k^2 - 2$, where k is a real constant.

    (a) Using the method of completing the square, find the coordinates of the vertex of the graph of $y = f(x)$ in terms of k. (2 marks)

    (b) On the same rectangular coordinate system, let D be the vertex of the graph of $y = f(x) + 4$ and E be the vertex of the graph of $y = f(-x) - 4$. Is there a point F on this rectangular coordinate system such that the coordinates of the circumcentre of ΔDEF are $(4, 0)$? Explain your answer. (4 marks)

Answers written in the margins will not be marked.

18. (a) *PQRS* is a quadrilateral cardboard, where $PQ = 18$ cm, $QR = 12$ cm, $RS = 15$ cm, $\angle QPS = 50^{\circ}$ and $\angle QRS = 70^{\circ}$. Find

    (i) the length of *QS*.

    (ii) $\angle PQS$.   (4 marks)

(b) The cardboard *PQRS* described in (a) is now folded along *QS* (see Figure 5). It is given that the angle between the plane *PQS* and *QRS* is 60°.
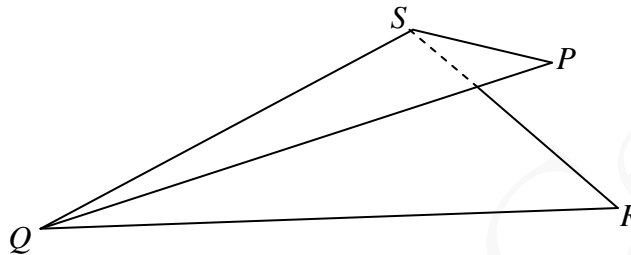

Figure 5

    (i) Find the shortest distance from *P* to the plane *QRS*.

    (ii) Let *X* be any point lying on the plane *PSQ*. Someone claims that the distance between *R* and *X* exceeds 9 cm. Is the claim correct? Explain your answer.   (4 marks)
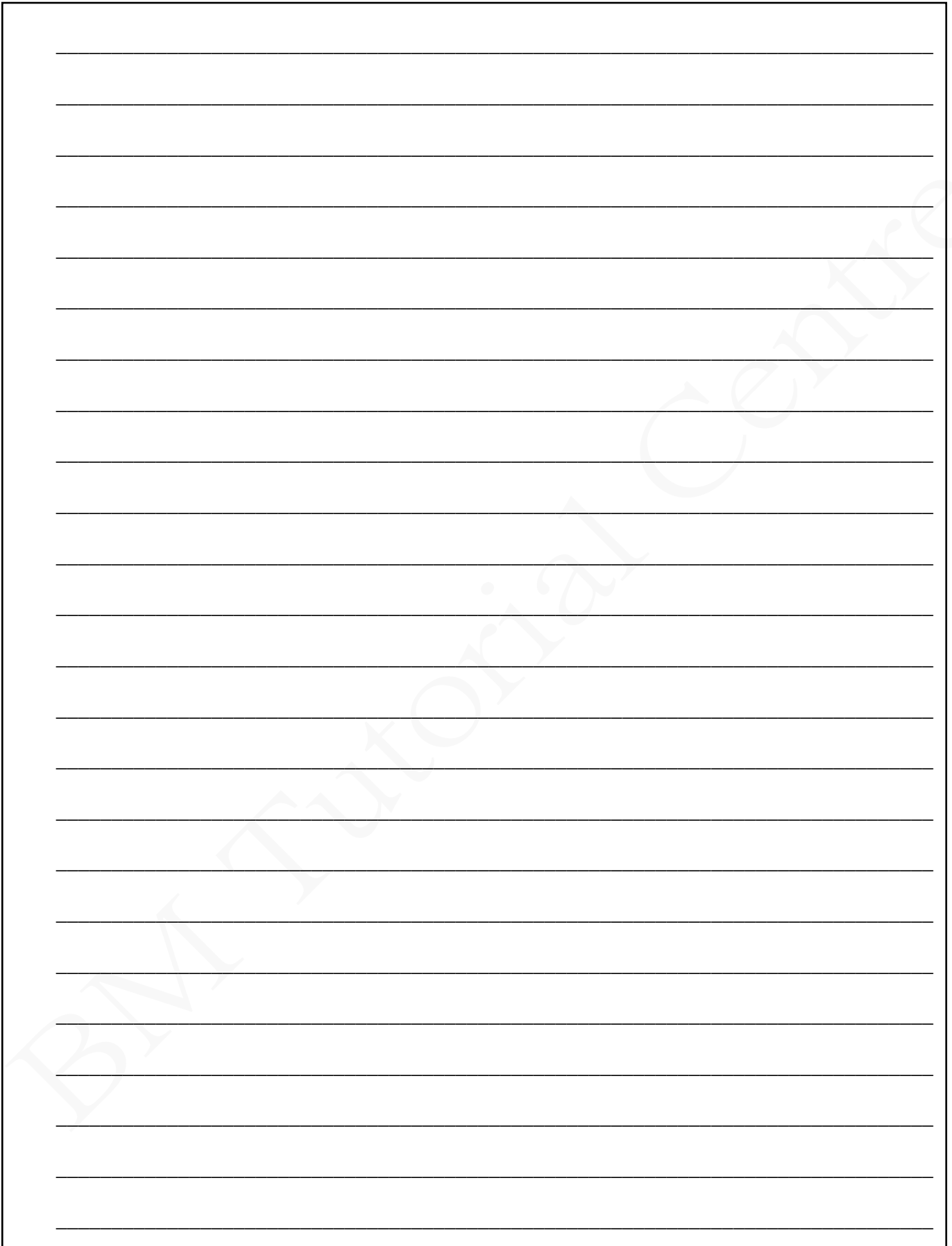
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19. Amy and Bob play a game consisting of two rounds. In the first round, they take turns to draw a card from a deck of 52 playing cards **with replacement**. The player who first gets an "ace" wins the first round. They play the first round until one of them wins. Amy draws first.

    (a) Find the probability that Bob wins the first round of the game. (3 marks)

    (b) In the second round of the game, more cards are drawn at a time. The player of this round adopts one of the following two options.

       Option 1: Two cards are drawn at a time. If the two cards have the same colour and face value (e.g. heart ace and diamond ace), then the player gets 100 tokens. If the cards have the same face value but different colours, then the player gets 50 tokens. Otherwise, the player gets no tokens.

       Option 2: Three cards are drawn at a time. If the three cards have the same face value, then the player gets 300 tokens. If two of the cards have the same colour and face value, then the player gets 30 tokens. If two of the cards have the same face value but different colours, then the player gets 15 tokens. Otherwise, the player gets no tokens.

       (i) If the player of the second round adopts Option 1, find the expected number of tokens got.

       (ii) Which option should the player of the second round adopt in order to maximize the expected number of tokens got? Explain your answer.

       (iii) Only the winner of the first round plays the second round. It is given that the player of the second round adopts the option which can maximize the expected number of tokens got. Bob claims that the probability of Amy getting no tokens in the game exceeds 0.9. Is the claim correct? Explain your answer. (10 marks)

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END OF PAPER