MOCK EXAM 5

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

Name: _____

(2 1/4 hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your name in the space provided on Page 1.

2. This paper consists of **THREE** sections, A(1), A(2), and B.

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your name on the graph paper and supplementary answer sheets.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

SECTION A(1) (35 marks)

1. Simplify $\frac{x^{-3}y^{5}}{(x^{2}y^{-3})^{-4}}$ and express your answer with positive indices. (3 marks)

2. Make b the subject of the formula $\frac{5-4b}{a}$ = 6. (3 marks)

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3. Factorize

(a) $9x^2 + 24xy + 16y^2$,

(b) $9x^2 + 24xy + 16y^2 - 25x^2y^2$. (3 marks)

4. The number of comic books Amy has is 5 times that of Bob has. If Amy gives Bob 8 comic books, then they have the same number of comic books. Find the total number of comic books they have. (4 marks)

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5. The marked price of a pair of shoes is 60% higher than its cost. The shoes are sold at a discount of 25% on its marked price and the profit is $72. Find the marked price of the shoes. (4 marks)

6. In a polar coordinate system, O is the pole. The polar coordinates of the points of A and B are $(18, 30^o)$ and $(18, 150^o)$ respectively. Let L be the axis of reflectional symmetry of ΔAOB.

   (a) Describe the geometric relationship between L and AB.

   (b) Find the polar coordinates of the point of intersection of L and AB. (4 marks)

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7. In Figure 1, O is the centre of the circle ABC. If $AB = AC$ and $\angle ABO = 36^o$, find $\angle BOC$.

(4 marks)

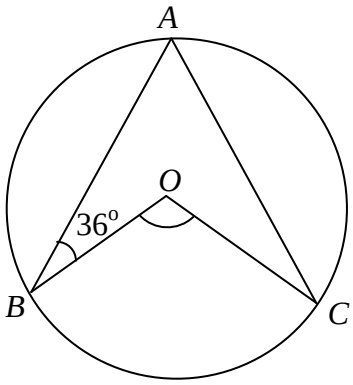

Figure 1

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8. A piece of string is measured as 20.2 m correct to the nearest 10 cm.

   (a) Find the least possible length of the string.

   (b) Someone claims that if the string is cut into pieces of string, each with a length of 20 cm correct to the nearest cm, at least 100 pieces of string must be obtained. Do you agree? Explain your answer. (5 marks)

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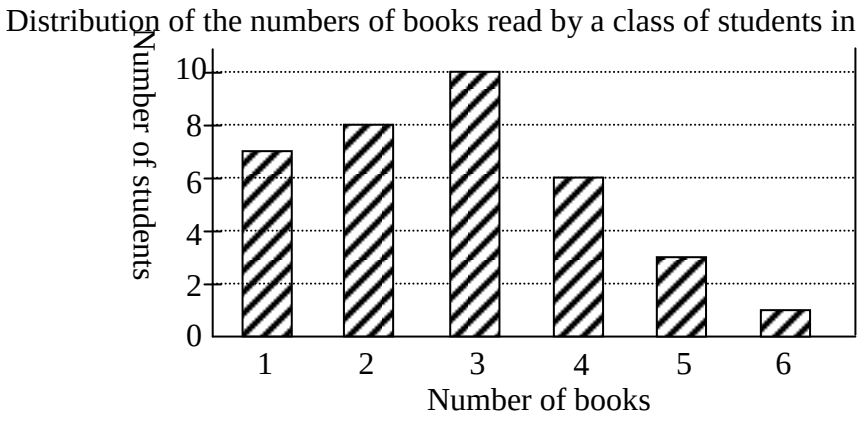
9. The bar chart below shows the distribution of the numbers of books read by a class of students in a month.



(a) Find the mean, the inter-quartile range and the standard deviation of the above distribution.

(b) A student was found to have reported the number of books she read in the month wrongly. She actually read 2 books instead of 5 books. Find the change in the standard deviation of the numbers of books read by the class of students in the month due to the correction of the datum.

(5 marks)

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SECTION A(2) (35 marks)

10. Let $\$C$ be the cost of publishing books. It is given that C is partly a constant and partly varies as the number of books(n) published. When $n = 75$, $C = 5\,900$; when $n = 100$, $C = 7\,700$.

    (a) Find the cost of publishing 90 books. (4 marks)

    (b) Can the cost of publishing books be \$7 880? Explain your answer. (2 marks)

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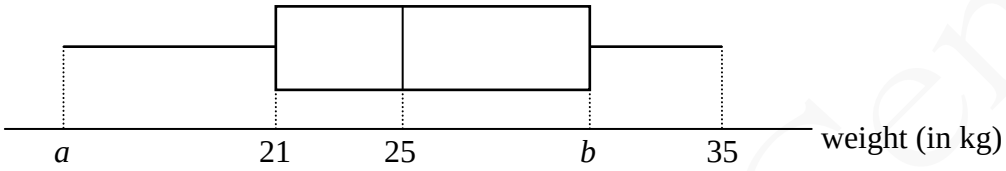
11. The stem-and-leaf diagram below shows the distribution of weights (in kg) of a group of children.

Stem (tens)	Leaf (units)
1	4 4 4 5 7 8 9
2	0 0 2 3 5
3	0 1 1 2

(a) Write down the median and the mode of the weights of the group of children. (2 marks)

(b) The box-and-whisker diagram below shows the distribution of the weights of another group of students. It is given that the range and the inter-quartile range of this distribution are 19 kg and 11 kg respectively.



(i) Find a and b.

(ii) For each group, a child is randomly selected. Someone claims that the probability that the sum of their weights is 45 kg or above is more than 0.5. Do you agree? Explain your answer. (4 marks)

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12. Figure 2(a) shows a solid right circular cone of base radius 48 cm and height 72 cm.

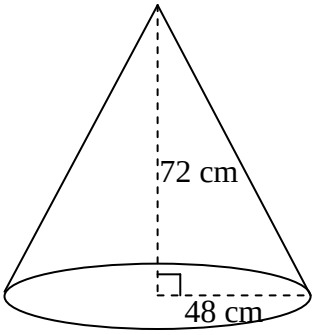

Figure 2(a)

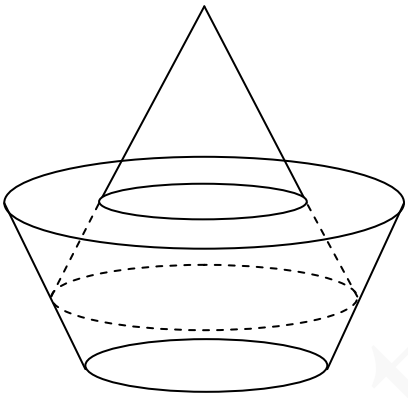

Figure 2(b)

(a) Find the volume of the circular cone in terms of π. (2 marks)

(b) A vessel in the form of a frustum is put on a horizontal surface. The radii of the upper surface and the lower surface are 60 cm and 36 cm respectively. Its height is 48 cm. The vessel is fully filled with water.

(i) Find the volume of the water in the vessel in terms of π.

(ii) The circular cone is now held vertically in the vessel as shown in Figure 2(b). A craftsman claims that the volume of the water remaining in the vessel is greater than 0.25 m^3. Do you agree? Explain your answer. (5 marks)

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13. The polynomial f(x) is divisible by $x - 2$. When f(x) is divided by $x^2 - x - 1$, the quotient and the remainder are $3x^2 + 16x + 14$ and $ax + a - 10$ respectively, where a is a constant.

(a) Find a. (3 marks)

(b) Prove that $x + 4$ is a factor of f(x). (1 mark)

(c) Someone claims that all the roots of the equation f(x) = 0 are real numbers. Is the claim correct? Explain your answer. (3 marks)

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14. The coordinates of the centre of the circle C are (5, –1). Denote the centre by P. It is given that C passes through $A(2, 3)$.

(a) Find the equation of C. (2 marks)

(b) L_1 is the tangent to C at A. Find the equation of L_1. (2 marks)

(c) L_1 cuts the x-axis at B. L_2 is another tangent drawn from B to the circle. L_2 touches C at D.

(i) Describe the geometric relationship between PB and $\angle ABD$.

(ii) Find the coordinates of D. (5 marks)

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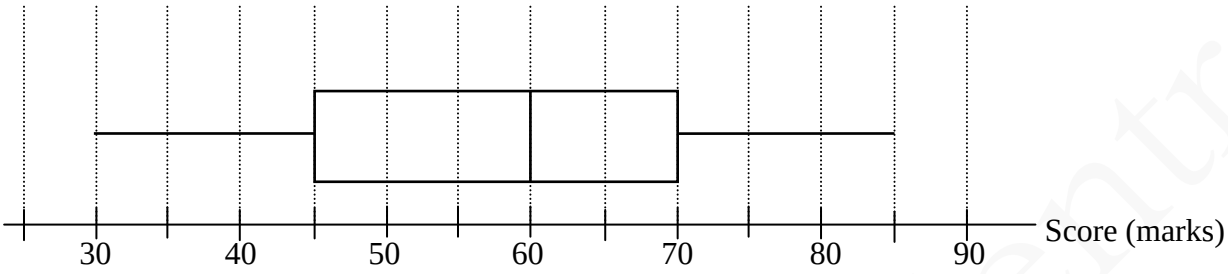
SECTION B (35 marks)

15. The box-and-whisker diagram below shows the distribution of the scores (in marks) of the students of a class in an exam. Amy gets the lowest score while John gets 51 marks in the exam. The standard scores of Amy and John in the exam are –5 and –2 respectively.



(a) Find the mean of the distribution. (2 marks)

(b) John claims that the standard scores of more than half of the students in the exam are negative. Do you agree? Explain your answer. (2 marks)

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16. A new theatre in the shape of a shell is built. The seats are arranged as shown in Figure 3. The first row has 11 seats. 2 more seats are added to the subsequent row. It is given that the theatre can only accommodate at most 1 344 seats. At most how many rows of seats can be installed? (4 marks)

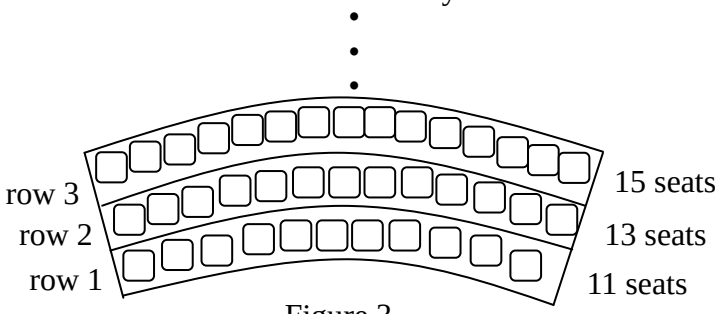

Figure 3

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17. (a) Let $f(x) = 108x - x^2$. Using the method of completing the square, find the coordinates of the vertex of the graph of $y = f(x)$. (2 marks)

(b) The length of a fence is 216 m. A farmer cuts the fence into 3 parts. One part is used to enclose an outdoor rectangular region of area A m^2 by using the wall of a greenhouse as one side. The other two parts are used to divide the region into three rectangular areas as shown in Figure 4.

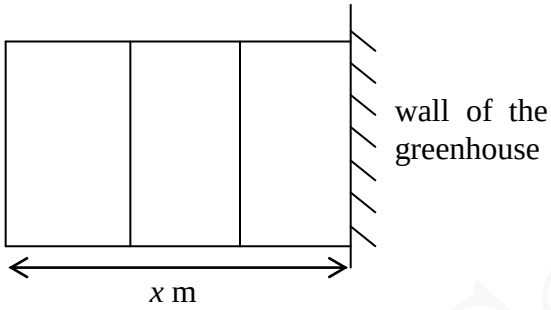

Figure 4

(i) Express A in terms of x.

(ii) The farmer claims that the area of the outdoor rectangular region can be greater than 2000 m^2. Do you agree? Explain your answer. (4 marks)

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18. Figure 5(a) shows a solid pyramid *ABCDE* with a rectangular base *ABCD*. *ABCD* stands vertically while *CDE* lies on a horizontal ground. It is given that *AD* = 20 cm, *DE* = 50 cm, *CE* = 60 cm and $\angle CED = 30^{\circ}$.

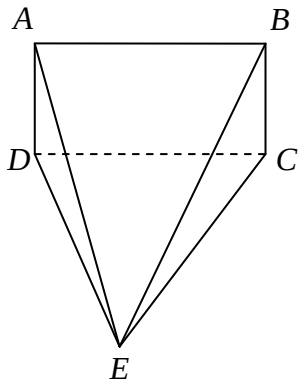

Figure 5(a)

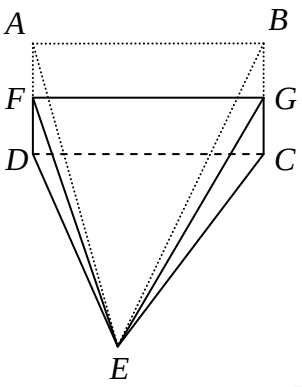

Figure 5(b)

(a) Find the length of *CD*. (2 marks)

(b) Find the angle between the plane *ABE* and the plane *DCE*. (3 marks)

(c) *F* and *G* are the mid-points of *AD* and *BC* respectively. A geometric model is made by cutting off *ABGFE* from *ABCDE* as shown in Figure 5(b). A craftsman claims that the area of Δ *EFG* is less than 800 cm^2. Do you agree? Explain your answer. (3 marks)

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19. Amy and Bob play a game consisting of two rounds. In the first round, they take turns to draw a card from ten cards numbered 1-10 with **replacement**. The player who first gets a multiple of 3 wins the first round. They play the first round until one of them wins. Amy draws first.

(a) Find the probability that Amy wins the first round of the game. (3 marks)

(b) In the second round of the game, balls are dropped one by one onto a slope. At the bottom of the slope, seven tubes are arranged side by side. A score is attached to each tube as shown in Figure 6. When a ball is dropped onto the slope, it falls randomly into one of the tubes. Each tube can hold at most three balls.

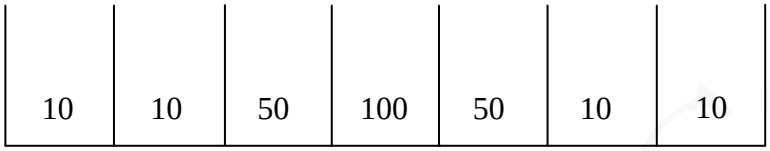

Figure 6

The player of this round adopts one of the following two options.

Option 1: Two balls are dropped one by one onto the slope. If the two balls fall into the same tube, then the player gets a score equal to 20 times the score attached to the tube. If the two balls fall into two adjacent tubes, then the player gets a score equal to the sum of the scores attached to the tubes. Otherwise, the player gets no score.

Option 2: Three balls are dropped one by one onto the slope. If the three balls fall into the same tube, then the player gets a score equal to the square of the score attached to the tube. If the three balls fall into three adjacent tubes, then the player gets a score equal to 10 times the sum of the scores attached to the tubes. Otherwise, the player gets no score.

(i) If the player of the second round adopts Option 1, find the expected score got.

(ii) Which option should the player of the second round adopt in order to maximize the score got? Explain your answer.

(iii) Only the winner of the first round plays the second round. It is given that the player of the second round adopts the option which can maximize the score got.

Bob claims that the probability of Amy getting no score in the game exceeds 0.9. Is the claim correct? Explain your answer.

(10 marks)

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