MOCK EXAM 7

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

Name: _____

(2 1/4 hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your name in the space provided on Page 1.
2. This paper consists of **THREE** sections, A(1), A(2), and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name on the graph paper and supplementary answer sheets.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

SECTION A(1) (35 marks)

1. Simplify $\frac{x^9 y^7}{(x^2 y^3)^4}$ and express your answer with positive indices. (3 marks)

2. Make y the subject of the formula $\frac{A-By}{C} = 5y$. (3 marks)

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3. (a) Round up 2059.856 to the nearest integer.

(b) Round down 2059.856 to 1 decimal place.

(c) Round off 2059.856 to 2 significant figures. (3 marks)

4. Simplify $\frac{3}{6m-5} - \frac{4}{8m-3}$. (3 marks)

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5. Factorize

(a) $6x - 6y$,

(b) $x^2 + 5xy - 6y^2$,

(c) $x^2 + 5xy - 6y^2 - 6x + 6y$. (4 marks)

6. (a) Find the range of values of x which satisfy both $\frac{2(x-4)}{7} + 15 > 3(2x - 3)$ and $x + 5 \geq 0$.

(b) How many positive integers satisfy both inequalities in (a)? (4 marks)

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7. The marked price of a watch is 60% above its cost. A gain of $30 is made by selling the watch at a discount of 30% on its marked price. Find the marked price of the watch. (5 marks)

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8. It is given that f(x) is the sum of two parts, one part varies as x and the other part varies as x^2. Suppose That f(4) = 44 and f(7) = 98.

   (a) Find f(x).

   (b) Solve the equation f(x) = 60. (5 marks)

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9. In Figure 1, *ABCDE* is a circle. It is given that *AB*//*EC* and *AB* = *AE*. *AC* and *BE* intersect at the point *F*.

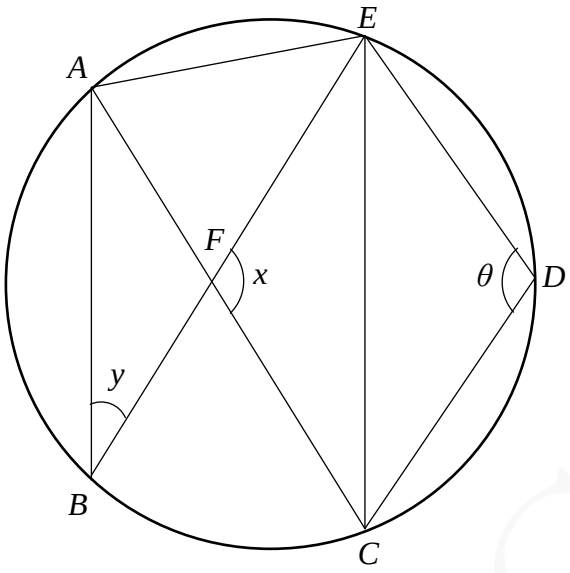

Figure 1

Express x and y in terms of θ. (5 marks)

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SECTION A(2) (35 marks)

10. The coordinates of the points A and B are (2, 9) and (10, 15) respectively. Let P be a moving point in The rectangular coordinate plane such that P is equidistant from A and B. Denote the locus of P by Γ.

    (a) Find the equation of Γ. (2 marks)

    (b) Γ intersects the x-axis and the y-axis at M and N respectively. Denote the origin by O. Let C be the circle which passes through O, M and N. Someone claims that the area of C exceeds 500 unit square. Is the claim correct? Explain your answer. (3 marks)

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11. The following table shows the distribution of the numbers of toys of some children:

Number of toys	0	1	2	3	4
Number of children	9	7	10	5	k

It is given that k is a positive integer.

(a) If the mode of the distribution is 2, write down

(i) the least possible value of k;

(ii) the greatest possible value of k. (2 marks)

(b) If the median of the distribution is 2, write down

(i) the least possible value of k;

(ii) the greatest possible value of k. (2 marks)

(c) If the mean of the distribution is 2, find the value of k. (2 marks)

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12. A solid metal cylinder of base radius 18 cm and height 9 cm is melted and recast into two similar solid cones. The ratio of the base area of the smaller cone to that of the larger cone is 1 : 4.

(a) Find the volume of the smaller cone in terms of π. (3 marks)

(b) If the height of the smaller cone is 12 cm, find the total surface area of the larger cone. (4 marks)

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13. In Figure 2, ABC is a triangle. D, E and F are points lying on BC such that $\angle ADB = \angle AEC$, $BD = EC$, and $BF = FC$.

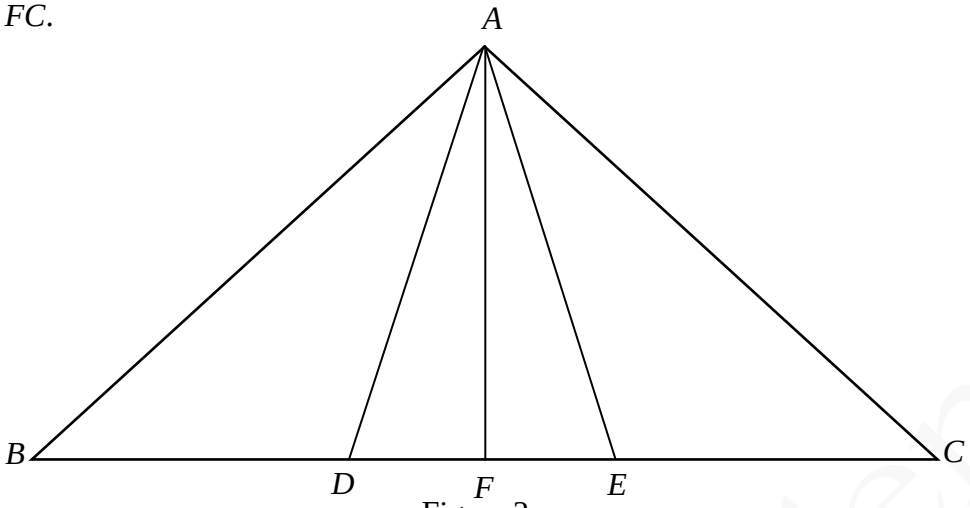

Figure 2

(a) Prove that $\Delta ABD \cong \Delta ACE$. (2 marks)

(b) Suppose that AD = 65 cm, DF = 25 cm and BE = 169 cm.

(i) Find AF.

(ii) Is ΔADC a right-angled triangle? Explain your answer. (5 marks)

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14. Let $f(x) = ax^4 + 2x^3 + bx^2 + cx - 9$, where a, b and c are constants. When $f(x)$ is divided by $x + 3$ and when $f(x)$ is divided by $x - 3$, the two remainders are equal.

It is given that $f(x) \equiv (\lambda x^2 + mx + 3)(12x^2 - 5x + n)$, where λ, m and n are constants.

(a) Find λ, m and n. (5 marks)

(b) How many real roots does the equation $f(x) = 0$ have? Explain your answer. (5 marks)

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SECTION B (35 marks)

15. If 6 girls and 8 boys randomly form a queue, find the probability that no girls are next to each other in the queue. (3 marks)

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16. The 1st term and the 38th term of an arithmetic sequence are –444 and –222 respectively. Find

(a) the common difference of the sequence, (2 marks)

(b) the greatest value of n such that the sum of the first n terms of the sequence is negative. (3 marks)

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17. (a) In Figure 3(a), *ABCD* is a paper card in the shape of a trapezium. It is given that $AD = DC = CB$, $AB = 40$ cm, $\angle ABD = 35^\circ$ and $\angle BAD = 70^\circ$.

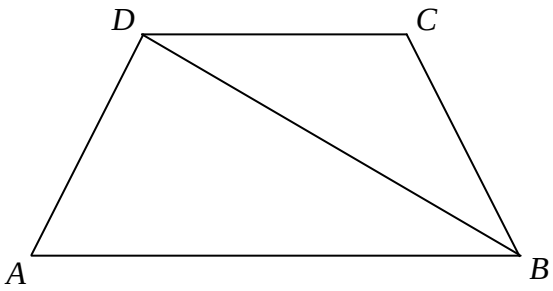

Figure 3(a)

Find the length of *AD*. (2 marks)

(b) The paper card in Figure 3(a) is folded along *BD* such that the distance between *A* and *C* is 30 cm (see Figure 3(b)).

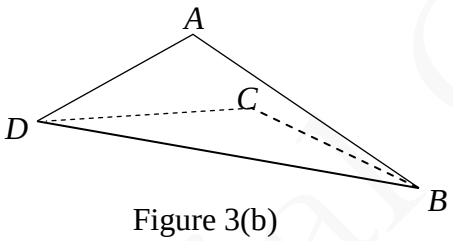

Figure 3(b)

(i) Find $\angle ABC$.

(ii) Find the angle between the plane *ABD* and the plane *BCD*. (5 marks)

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18. The equation of the parabola Γ is $y = 2x^2 + 4kx - 2x + 6k - 5$, where k is a real constant. Denote the straight line $y = 8$ by L.

(a) Prove that L and Γ intersect at two distinct points. (3 marks)

(b) The points of intersection of L and Γ are A and B.

(i) Let a and b be the x-coordinates of A and B respectively. Prove that $(a - b)^2 = 4k^2 - 16k + 27$.

(ii) Is it possible that the distance between A and B is less than 3? Explain your answer.

(5 marks)

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19. Let $f(x) = x^2 - 4kx - 12x + 4k^2 + 18k + 44$, where k is positive constant. Denote the vertex of the graph of $y = f(x)$ and the vertex of the graph $y = -f(x) + 16$ by Q and R respectively.

(a) Using the method of completing the square, express the coordinates of Q in terms of k. (2 marks)

(b) Write down the coordinates of R in terms of k. (1 mark)

(c) The coordinates of the point S are $(10k + 6, 8)$. Denote the inscribed circle of ΔQRS by C. Denote the centre of C by U. Suppose that QS is the tangent to C at the point T.

(i) Express the equation of the straight line which passes through Q and S in terms of k.

(ii) Express the equation of C in terms of k.

(iii) It is given that the coordinates of the point V are (47, 20). Is it possible that $STUV$ is a rectangle? Explain your answer. (9 marks)

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END OF PAPER

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