MOCK EXAM 8

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

Name: _____

($2\frac{1}{4}$ hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your name in the space provided on Page 1.
2. This paper consists of **THREE** sections, A(1), A(2), and B.
3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name on the graph paper and supplementary answer sheets.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

SECTION A(1) (35 marks)

1. Make b the subject of the formula $\frac{a-3}{5} = \frac{2b+3}{2}$. (3 marks)

2. Simplify $\frac{(m^{-2}n^{-3})^2}{(m^{-1})^{-4}}$ and express your answer with positive indices. (3 marks)

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3. Factorize

(a) $12x^2 - 5xy - 3y^2$,

(b) $6x + 2y - 12x^2 + 5xy + 3y^2$. (3 marks)

4. There are only two kinds of tickets for a train: adult tickets and child tickets. The prices of an adult ticket and a child ticket are \$88 and \$48 respectively. For a certain journey, the number of adult tickets sold is 8 times the number of child tickets sold and the sum of money for the train tickets sold is \$57 152. Find the total number of train tickets sold for that journey. (4 marks)

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5. (a) Solve the inequality $2-\frac{x-3}{5}<7-x$.

(b) Find all integers satisfying both the inequalities $2-\frac{x-3}{5}<7-x$ and $3x-8\geq 4$.

(4 marks)

6. In a polar coordinate system, O is the pole. The polar coordinates of the points A and B are $(5, 87^{o})$ and $(12, 177^{o})$ respectively.

(a) Find $\angle AOB$.

(b) Find the perimeter of $\triangle AOB$. (4 marks)

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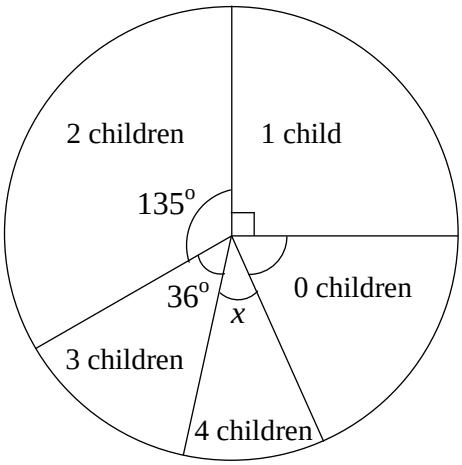
7. The pie chart below shows the distribution of the number of children in families living in a building.



Distribution of the number of children in families living in the building

If a family is selected from the family living in the building randomly, the probability that the selected family having 4 children is 0.075.

(a) Find x.

(b) There are 32 families having no children. Find the total number of families. (4 marks)

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8. It is given that y varies inversely as x^2. When $x = 8$, $y = 2$.

(a) Express y in terms of x.

(b) If the value of x is increased from 4 to 16, find the change in the value of y . (5 marks)

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9. A packet of eggs is termed as *standard* if its weight is measured as 800 g correct to the nearest 20 g.

   (a) Find the greatest possible weight of a *standard* packet of eggs.

   (b) Someone claims that the total weight of 180 *standard* packets of eggs can be measured as 146 kg correct to the nearest 0.2 kg. Do you agree? Explain your answer. (5 marks)

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SECTION A(2) (35 marks)

10. An inverted right circular conical vessel contains some wine. It is held vertically. When 3488π cm^3 of wine is poured into the vessel without overflowing, the depth of wine is found to increase by 40%.

    (a) Express the final volume of wine in the vessel in terms of π. (3 marks)

    (b) The final depth of wine is 21 cm. Someone claims that the original area of the wet curved surface of the vessel exceeds 1 600 cm^2. Do you agree? Explain your answer. (3 marks)

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11. The stem-and-leaf diagram below shows the distribution of the numbers of weekly hours spent by a group of students online:

Stem (tens)	Leaf (units)							
0	7	7	9	9	9			
1	1	2	2	*a*	6	7	8	8
2	2	*b*						

It is given that the mean and the range of the above distribution are 14 hours and 21 hours respectively.

(a) Find the inter-quartile range and the standard deviation of the above distribution. (5 marks)

(b) If a student is randomly selected from the group, find the probability that the number of hours spent by the student online exceeds 15 hours. (2 marks)

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12. The equation of the circle C is $x^2 + y^2 - 36x - 48y + 611 = 0$. Let G be the centre of C. Denote the origin by O.

(a) Find OG. (2 marks)

(b) Does O lie inside C? Explain your answer. (1 marks)

(c) Let P be a moving point in the rectangular coordinate plane such that $OP = GP$. Denote the locus of P by Γ. Suppose that Γ cuts C at the points A and B. Find the area of the quadrilateral $OAGB$. (4 marks)

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13. Let $f(x) = 2x(x + 1)^2 + ax + b$, where a and b are constants. It is given that $x - 2$ is a factor of $f(x)$. When $f(x)$ is divided by $x + 3$, the remainder is $a - 5b$.

(a) Find a and b. (3 marks)

(b) Someone claims that the equation $f(x) = 0$ has two irrational roots. Do you agree? Explain your answer. (4 marks)

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14. In Figure 1, *ABCD* is a trapezium with $\angle BCD = 90^o$ and *BA*//*CD*. *E* is a point lying on *BD* such that $\angle AED = 90^o$.

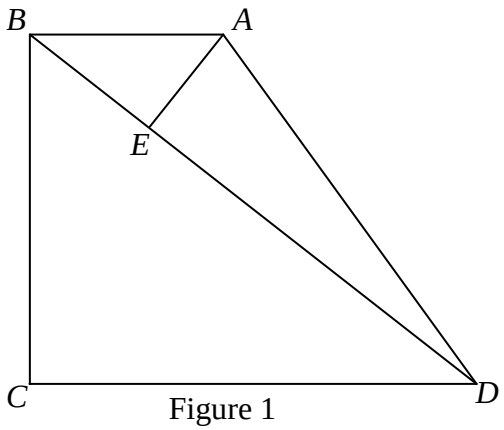

Figure 1

(a) Prove that $\triangle ABE \sim \triangle BDC$. (2 marks)

(b) It is given that *AB* = 20 cm, *AE* = 12 cm and *ED* = 34 cm.

(i) Find the length of *CD*.

(ii) Find the area of $\triangle BDC$.

(iii) Is there a point *F* lying on *BD* such that the distance between *C* and *F* is less than 24 cm? Explain your answer. (6 marks)

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SECTION B (35 marks)

15. An eight-digit phone number is formed by a permutation of 0-9. It is given that 0 and 1 cannot be the first digit of a phone number.

    (a) How many different eight-digit phone numbers can be formed? (1 mark)

    (b) If the first and the last digit of an eight-digit phone number are even numbers, how many different eight-digit phone numbers can be formed? (2 marks)

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16. In a test, the mean of the distribution of the scores of a class of students is 65 marks. The standard score of Amy and Billy are 2.2 and –2.9 respectively. Amy gets 87 marks. A teacher claims that the range of the distribution is at most 50 marks. Is the claim correct? Explain your answer. (3 marks)

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17. Let a and b be constants. Denote the graph of $y = a + \log_b x$ by G. The x-intercept of G is 25 and G passes through the point $(\frac{1}{625}, -6)$. Express x in terms of y. (4 marks)

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18. A certain vaccine is produced to prevent a disease. It is given that the amount of vaccines produced in the 1st year since its production is 4.6×10^7 and in subsequent years the amount of production is 8% less than that produced in the previous year.

(a) Find the total amount of vaccines produced in the first 10 years since its production. (2 marks)

(b) Someone claims that the total amount of vaccines produced since its production will not exceed 5.8×10^8. Do you agree? Explain your answer. (2 marks)

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19. It is given that f(x) partly varies as x^2 and partly varies as x. Suppose that f(4) = –64 and f(10) = –40.

(a) Find f(x). (3 marks)

(b) Let Q be the vertex of the graph of y = f(x) and R be the vertex of the graph of y = f(x + 60) + 84.

(i) Using the method of completing the square, find the coordinates of Q.

(ii) Write down the coordinates of R.

(iii) The coordinates of the point S are (16, 62). Let P be the circumcenter of $\triangle QRS$. Someone claims that P lies inside $\triangle QRS$. Do you agree? Explain your answer. (5 marks)

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20. ABC is a thin triangular metal sheet where $BC = 30$ cm, $\angle BAC = 40^o$ and $\angle ABC = 95^o$.

(a) Find the length of AB. (2 marks)

(b) In Figure 2, the thin metal sheet ABC is held such that only the vertex C lies on the horizontal ground. D and E are points lying on the horizontal ground vertically below the vertices A and B respectively. AB produced meets the horizontal ground at the point F. A craftsman finds that $AD = 12$ cm and $BE = 3$ cm.

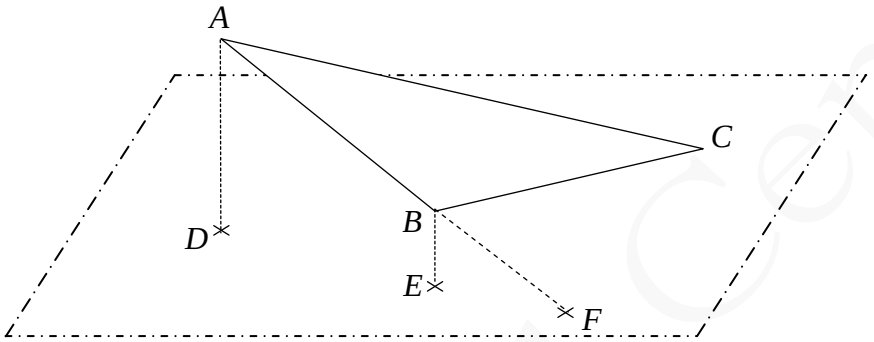

Figure 2

(i) Find the distance between B and F.

(ii) Find the area of $\triangle ACF$.

(iii) Find the inclination of the thin metal sheet ABC to the horizontal ground.

(iv) The craftsman claims that the area of $\triangle CDF$ is greater than 660 cm^2. Do you agree? Explain your answer. (11 marks)

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END OF PAPER

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