Suggested Solution for 2012 HKDSE Mathematics(core) Multiple Choice Questions

1. C

$\frac{(2x^4)^3}{2x^5}$

$= \frac{2^3x^{4\times 3}}{2x^5}$

$= \frac{2^3x^{12}}{2x^5}$

$= 4x^7$

2. D

$(4x + y)^2 - (4x - y)^2$

$= (4x + y + 4x - y)[4x + y - (4x - y)]$ $[a^2 - b^2 = (a + b)(a - b)]$

$= 8x(4x + y - 4x + y)$

$= 8x(2y)$

$= 16xy$

3. C

R.H.S. $= x^2 + (2 + q)x + 2q + 10$

By comparing the coefficients of x,

$2 + q = 0$

$q = -2$

By comparing the constant terms,

$p = 2(-2) + 10$

$= 6$

Alternatively

Substitute $x = -2$ to both sides of the identity,

$(-2)^2 + p = (-2 + 2)(-2 + q) + 10$

$4 + p = 10$

$p = 6$

4. B

Let $f(x) = x^3 + 4x^2 + kx - 12$.

By Factor Theorem,

$f(-3) = 0$

$(-3)^3 + 4(-3)^2 + k(-3) - 12 = 0$

$k = -1$

5. B

$\begin{cases} m + 2n + 6 = 7 \text{ i.e. } m + 2n = 1 \ldots (1) \\ 2m - n = 7 \ldots (2) \end{cases}$

$(1) \times 2 - (2)$,

$5n = -5$

$n = -1$

Alternatively

From (1), $m = 1 - 2n \ldots (3)$

Substitute (3) into (2),

$2(1 - 2n) - n = 7$

$-5n = 5$

$n = -1$

6. D

The graph opens downwards.

$\therefore \quad a < 0$

The equation of the axis of symmetry of the graph is $x = -b > 0$.

$\therefore \quad b < 0$

7. C

$15 + 4x < 3$ or $9 - 2x > 1$

$4x < -12$ or $8 > 2x$

$x < -3$ or $x < 4$

$\therefore \quad x < 4$

8. D

Percentage required

= 37.5% × 80% + (1 – 37.5%) × 60%

= 67.5%

9. A

$\frac{6x+5y}{3y-2x} = 7$

$6x + 5y = 7(3y - 2x)$

$6x + 5y = 21y - 14x$

$20x = 16y$

$\frac{x}{y} = \frac{16}{20} = \frac{4}{5}$ i.e. $x : y = 4 : 5$

10. D

Let $y = kx^2 + \frac{k_1}{x}$ where k and k_1 are constants. Then,

$-4 = k(1)^2 + \frac{k_1}{1}$ i.e. $k + k_1 = -4$ … (1)

$5 = k(2)^2 + \frac{k_1}{2}$ i.e. $8k + k_1 = 10$ … (2)

(2) – (1),

$7k = 14$

$k = 2$

and $k_1 = -6$

When $x = -2$,

$y = 2(-2)^2 + \frac{-6}{-2}$

$= 11$

11. C

Her average typing speed

$= \frac{3\times63+4\times56}{7}$

= 59 words per minute

12. B

Let T(n) be the number of dots in the nth pattern. Then,

T(1) = 1

T(2) = 1 + 1 = 2

T(3) = 2 + 2 = 4

T(4) = 4 + 3 = 7

T(5) = 7 + 4 = 11

T(6) = 11 + 5 = 16

T(7) = 16 + 6 = 22

T(8) = 22 + 7 = 29

13. D

0.0322515

= 0.0323 (correct to 3 significant figures)

= 0.0323 (correct to 4 decimal places)

= 0.032252 (correct to 5 significant figures)

= 0.032252 (correct to 6 decimal places)

14. B

$\frac{(25-0.5)m}{(5+0.5)cm} \leq n < \frac{(25+0.5)m}{(5-0.5)cm}$

$\frac{(25-0.5)\times100cm}{(5+0.5)cm} \leq n < \frac{(25+0.5)\times100cm}{(5-0.5)cm}$

$445\frac{5}{11} \leq n < 566\frac{2}{3}$

$\therefore$ The greatest possible value of n is 566.

15. A

By Pythagoras' theorem,

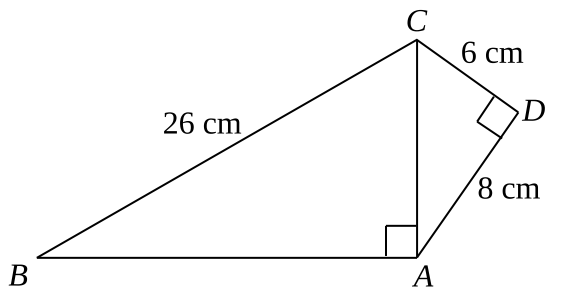
$AC^2 = AD^2 + CD^2 = 6^2 + 8^2 = 100$ i.e. $AC = 10$ cm

$AB^2 + AC^2 = BC^2$

$AB^2 + 100 = 26^2$

$AB = 24$ cm

Area of $ABCD = \frac{1}{2} \times 6 \times 8 + \frac{1}{2} \times 24 \times 10 = 144$ cm^2



16. B

$\overset{\frown}{AB} = 2\pi \times OA \times \frac{\angle AOB}{360^o}$

$12\pi = 2\pi \times 30 \times \frac{\angle AOB}{360^o}$

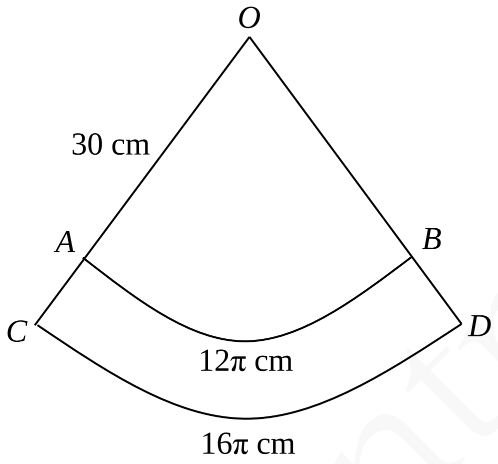
$\angle AOB = 72^o$

$\overset{\frown}{CD} = 2\pi \times OC \times \frac{\angle AOB}{360^o}$

$16\pi = 2\pi \times OC \times \frac{72^o}{360^o}$

$OC = 40$ cm

$AC = OC - OA = 40 - 30 = 10$ cm



17. B

Note that $\triangle GDF \sim \triangle GAE$.

$\because$ $GD : GA = 1 : 2$

$\frac{Area\ of\ \Delta GDF}{Area\ of\ \Delta GAE} = \left(\frac{GD}{GA}\right)^2$

$\frac{3}{Area\ of\ \Delta GAE} = \left(\frac{1}{2}\right)^2$

Area of $\triangle GAE = 12$ cm^2

Area of $DAEF = 12 - 3 = 9$ cm^2

Draw FH such that $DA // FH$.

Note that $\triangle GDF \cong \triangle FHE$.

Area of $\triangle FHE$ = area of $\triangle GDF = 3$ cm^2

Area of parallelogram $DAHF = 9 - 3 = 6$ cm^2

Area of $DAHF$: area of $FHBC = DF : FC$

6 : area of $FHBC = 3 : 4$

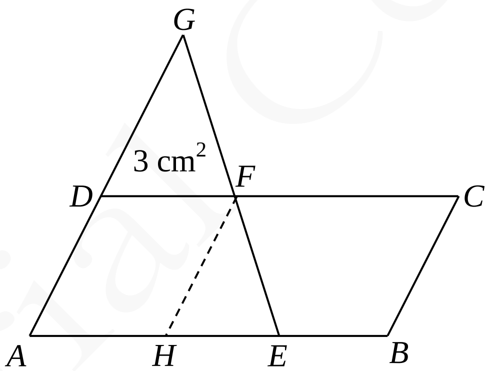
Area of $FHBC = 8$ cm^2

Area of $ABCD$

= Area of $DAHF$ + area of $FHBC$

= 6 + 8

= 14 cm^2



Alternatively

$\because$ $DF : FC = 3 : 4$ and $DF : AE = 1 : 2 = 3 : 6$

$\therefore$ $AE : EB = 6 : 1$

Area of $DAEF$: area of $FEBC = (3 + 6) : (4 + 1)$

[$\because$ $DAEF$ and $FEBC$ have the same height.]

9 : area of $FEBC = 9 : 5$

Area of $FEBC = 5$ cm^2

Area of $ABCD$

= Area of $DAEF$ + area of $FEBC$

= 9 + 5

= 14 cm^2

Alternatively

Construct a point H such that $CDGH$ is a parallelogram.

$\because$ $GD : DA = 1 : 1$

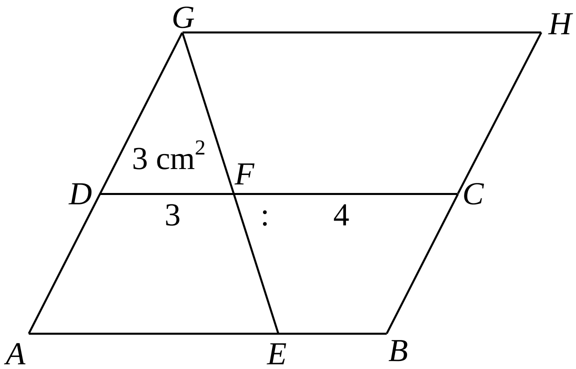
$\therefore$ Area of $ABCD$ = area of $CDGH$

$\because$ $\triangle GDF$ and $FCHG$ have the same height.

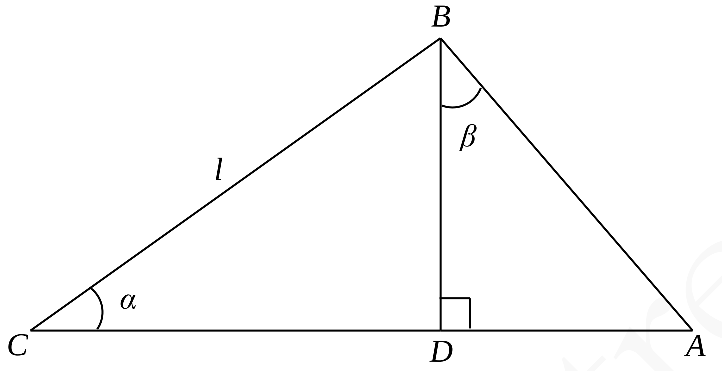
$\therefore$ Area of $FCHG$: area of $\triangle GDF = (4 + 7) : 3$

➔ Area of $FCHG = 11$ cm^2

$\therefore$ Area of $CDGH = 3 + 11 = 14$ cm^2 i.e. Area of $ABCD = 14$ cm^2



18. A



$\frac{BD}{l} = \sin\alpha$

$BD = l\sin\alpha$

$\frac{BD}{AB} = \cos\beta$

$AB = \frac{BD}{\cos\beta}$

$= \frac{l\sin\alpha}{\cos\beta}$

19. C

$\frac{\cos 60^o}{1-\cos(90^o-\theta)} + \frac{\cos 240^o}{1-\cos(270^o-\theta)}$

$= \frac{0.5}{1-\sin\theta} + \frac{-0.5}{1-(-\sin\theta)}$

$= \frac{0.5(1+\sin\theta)-0.5(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{0.5+0.5\sin\theta-0.5+0.5\sin\theta}{1-\sin^2\theta}$

$= \frac{\sin\theta}{\cos^2\theta}$

$= \frac{\tan\theta}{\cos\theta}$

20. C

Join AD.

$\angle OAD = \angle ODA$ (base $\angle$, isos. $\triangle$)

$\angle BAD + \angle BCD = 180^o$ (opp. $\angle$s, cyclic quad.)

$\angle OAD + \angle BAO + \angle BCD = 180^o$

$\angle OAD + 28^o + 114^o = 180^o$

$\angle OAD = 38^o$

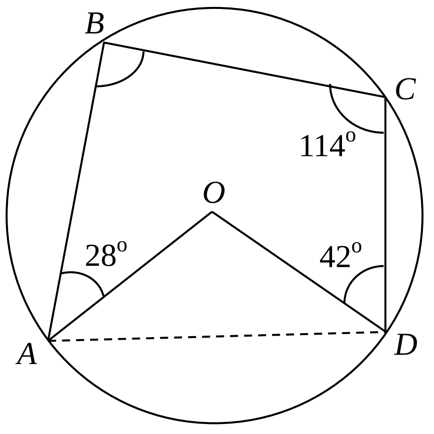
$\therefore \angle ODA = \angle OAD = 38^o$

$\angle ABC + \angle ADC = 180^o$ (opp. $\angle$s, cyclic quad.)

$\angle ABC + \angle ODA + \angle CDO = 180^o$

$\angle ABC + 38^o + 42^o = 180^o$

$\angle ABC = 100^o$



21. D

Let O be the centre of the circle. Then,

Radius = 12 ÷ 2 = 6 cm and $OA = OB = OC = OD = CD = 6$ cm

∴ $\triangle COD$ is an equilateral △ with $\angle COD = 60^{o}$.

Area of $\triangle COD$

$= \frac{1}{2} \times 6 \times 6 \times \sin 60^{o}$

$= 9\sqrt{3}$ cm^2

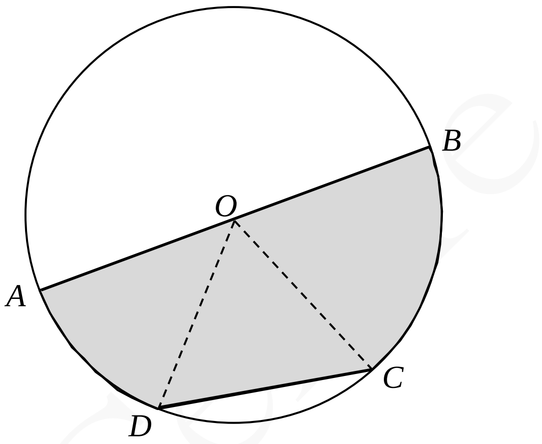
Note that $\angle AOD + \angle BOC = 120^{o}$

∴ Area of sector AOD + area of sector BOC

$= \pi(6)^2 \times \frac{120^o}{360^o}$

$= 12\pi$ cm^2

∴ Area of the shaded region = $(12\pi + 9\sqrt{3})$ cm^2



22. A

Each exterior angle

$= \frac{360^o}{12}$

$= 30^{o}$

∴ I is true.

Each interior angle

$= 180^{o} - 30^{o}$

$= 150^{o}$

∴ II is true.

∵ $n = 12$

∴ The number of axes of reflectional symmetry of the polygon is 12.

∴ III is NOT true.

23. D

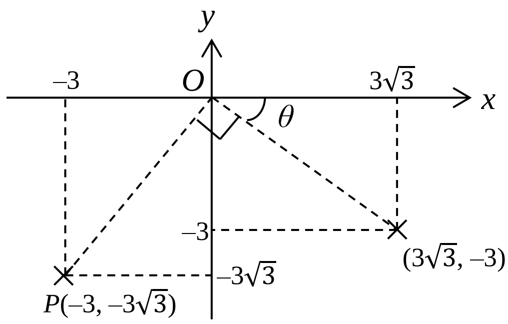
The rectangular coordinates of P's image are $(3\sqrt{3}, -3)$.

$r = \sqrt{(3\sqrt{3})^2 + (-3)^2} = 6$

$\tan\theta = \frac{3}{3\sqrt{3}}$

$\theta = 30^{o}$

The polar coordinates of its image are $(6, 360^{o} - 30^{o})$ i.e. $(6, 330^{o})$.



24. A

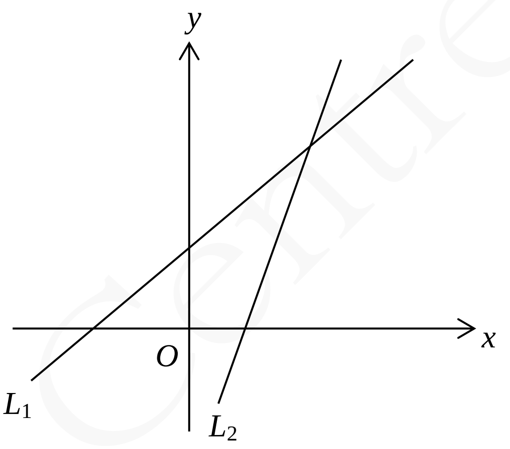
Let the coordinates of P be (x, y).

$\sqrt{(x-20)^2+(y-12)^2} = 5$

$(x-20)^2 + (y-12)^2 = 5^2$

The locus of P is a circle with centre (20, 12) and radius 5.

25. C



Rewrite $L_1 : y = -ax + b$ and $L_2 : y = -cx + d$.

Then, for L_1, slope = $-a$, y-intercept = b and x-intercept = $\frac{b}{a}$.

For L_2, slope = $-c$, y-intercept = d and x-intercept = $\frac{d}{c}$.

From the figure, the slope of $L_1 = -a > 0$. i.e. $a < 0$

∴ I is true.

From the figure, the slope of L_2 > the slope of L_1.

∴ $-c > -a$ i.e. $c < a$

∴ II is NOT true.

From the figure, y-intercept of L_1 > y-intercept of L_2.

∴ $b > d$

∴ III is true.

From the figure, x-intercept of L_2 > x-intercept of L_1.

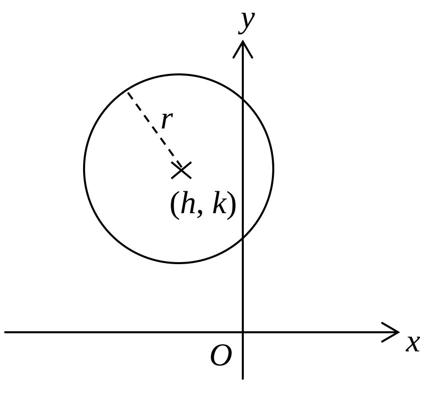
∴ $\frac{d}{c} > \frac{b}{a}$

$\frac{d}{c} \times ac > \frac{b}{a} \times ac$ [∵ $c < a < 0$]

i.e. $ad > bc$

∴ IV is true.

26. A



From the figure, $k > -h$ i.e. $h + k > 0$

∴ I is true.

From the figure, $h < 0$.

∴ $r - h > 0$

∴ II is true.

From the figure, $k > r$. i.e. $r - k < 0$

∴ III is NOT true.

27. A

The 3-digit number is divisible by 5 only if ◆ is 0 or 5 and ★ can be any integer from 0 to 9 inclusive.

The required probability

$= \frac{2\times 10}{10\times 10}$

$= \frac{1}{5}$

28. B

Out of the 20 members, there are 6 members who are not under the age of 74.

The required probability

$= \frac{6}{20}$

= 0.3

29. B

Mean = $\frac{1\times 8+2\times 4+3\times 6+4\times 2}{2+8+4+6+2}$

$= \frac{21}{11}$

Standard deviation

$= \sqrt{\frac{2\left(0-\frac{21}{11}\right)^2+8\left(1-\frac{21}{11}\right)^2+4\left(2-\frac{21}{11}\right)^2+6\left(3-\frac{21}{11}\right)^2+2\left(4-\frac{21}{11}\right)^2}{2+8+4+6+2}}$

≈ 1.16 (correct to 2 decimal places)

30. D

∵ Median = 14

∴ $m \geq 14$ and $n \geq 14$

∴ I is true.

Mean = $\frac{19+10+12\times 2+13\times 2+14+15+16+m+n}{11} = 14$

∴ $m + n = 30$

∴ III is true.

∵ $m + n = 30$ and $m \geq 14$

∴ $n = 30 - m \leq 30 - 14 = 16$

∴ II is true.

31. B

a in the H.C.F. must come from the 3rd expression.

b^5 in the L.C.M. must come from the 3rd expression.

$\therefore$ The 3rd expression is ab^5.

32. C

$y = mn^x$

$\log_3 y = \log_3(mn^x)$

$\log_3 y = \log_3 m + \log_3 n^x$

$= (\log_3 n)x + \log_3 m$

From the figure, the slope of the graph = 2.

$\therefore$ slope $= \log_3 n = 2$

$n = 3^2 = 9$

<u>Alternatively</u>

From the figure, the slope of the graph = 2 and the $\log_3 y$-intercept = 4.

Then, $\log_3 y = 2x + 4$

$y = 3^{2x+4}$

$y = 3^4(3^2)^x$

$y = 81(9)^x$

$\therefore$ $n = 9$

33. A

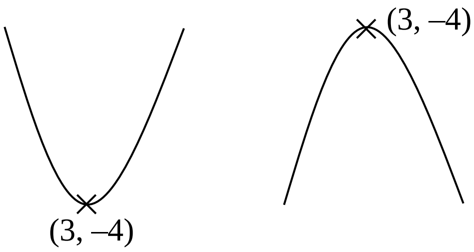
$AD0000002012_{16}$

$= 10 \times 16^{11} + 13 \times 16^{10} + 2 \times 16^3 + 1 \times 16 + 2$

$= 10(16^{11}) + 13(16^{10}) + 8\ 210$

16^{11}	16^{10}	16^9	16^8	16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0
A	D	0	0	0	0	0	0	2	0	1	2

34. C



$\because$ (3, –4) is the vertex of the graph.

$\therefore$ $f(3) = -4$ i.e. $f(3) + 4 = 0$

$\therefore$ 3 is a root of the equation $f(x) + 4 = 0$.

$\therefore$ The roots of the equation $f(x) + 4 = 0$ are real numbers.

Note that the graph may open upwards or downwards with the vertex (3, –4).

The equation $f(x) = 0$ has no real roots if the graph opens downwards. $\therefore$ A is not true.

The equation $f(x) - 3 = 0$ has no real roots if the graph opens downwards. $\therefore$ B is not true.

The equation $f(x) + 5 = 0$ has real roots if the graph opens downwards. $\therefore$ D is not true.

35. A

$i^3(\beta i - 3)$

$= \beta i^4 - 3i^3$

$= \beta - 3(-i)$

$= \beta + 3i$

36. D

Using (0, 0) as a testing point, it is not difficult to see that the constraints for the shaded region are:

$$\begin{cases} y \geq 3 \\ y \leq x + 3 \\ y \leq 6 - 2x \end{cases}$$

As (h, k) lies in the region,

$k \geq 3$

$k \leq h + 3$ i.e. $h - k \geq -3$

$k \leq 6 - 2h$ i.e. $2h + k \leq 6$

$\therefore$ I, II and III are true.

37. A

Let d be the common difference.

$a_{18} = a_1 + 17d = 26 \ldots (1)$

$a_{23} = a_1 + 22d = 61 \ldots (2)$

(2) – (1),

$5d = 35$

$d = 7 \ldots (3)$

Substitute (3) into (1),

$a_1 + 17(7) = 26$

$a_1 = -93$

$a_{14} = a_1 + 13d$

$= -93 + 13(7)$

$= -2$

< 0

$\therefore$ I is true.

$a_1 - a_2 = -d$

$= -7$

< 0

$\therefore$ II is true.

$a_1 + a_2 + a_3 + \ldots + a_{27}$

$= \frac{(2a_1+26d)\times 27}{2}$

$= \frac{[2(-93)+26(7)]\times 27}{2}$

$= -54$

< 0

$\therefore$ III is NOT true.

38. C

f$(x-2)+1$ is a translation of f(x) by 2 units to the right followed by a translation of 1 unit upwards.

39. D

Let $y = h + 3\cos kx^{\circ}$, where h and k are constants.

From the figure, when $x = 0$, $y = 7$,

$7 = h + 3\cos k(0)^{\circ}$

$h = 4$

When $x = 90$, $y = 1$,

$1 = 4 + 3\cos k(90)^{\circ}$

$\cos k(90)^{\circ} = -1$

$k(90)^{\circ} = 180^{\circ}$

$k = 2$

$\therefore \quad y = 4 + 3\cos 2x^{\circ}$

<u>Alternatively</u>

Comparing with $y = \cos x$, the graph is stretched 3 times in y-direction and translated 4 units upwards. In x-direction, it is compressed 2 times.

$\therefore \quad h = 4$ and $k = 2$.

$\therefore \quad y = 4 + 3\cos 2x^{\circ}$

40. D

Let E be the foot of the perpendicular from A to BC. Then, E is also the foot of the perpendicular from D to BC.

The required angle is $\angle AED(\theta)$.

Let the length of one side of the tetrahedron be $2x$.

By Pythagoras' theorem,

$AE^2 + EC^2 = AC^2$

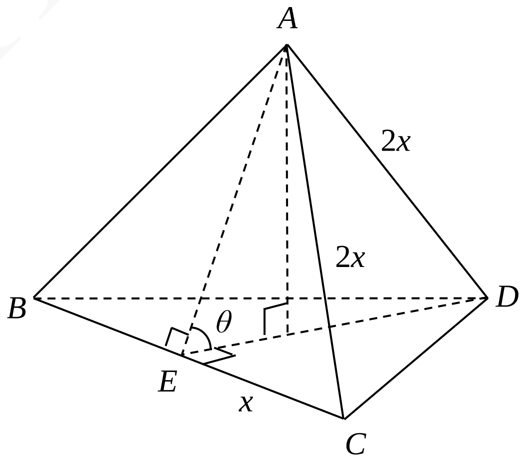
$AE^2 + x^2 = (2x)^2$

$AE^2 = 3x^2$

$AE = \sqrt{3}x$

Note that $DE = AE = \sqrt{3}x$.

By cosine formula,



$\cos\theta = \frac{AE^2+DE^2-AD^2}{2(AE)(DE)}$

$= \frac{(\sqrt{3}x)^2+(\sqrt{3}x)^2-(2x)^2}{2(\sqrt{3}x)(\sqrt{3}x)}$

$= \frac{1}{3}$

$\theta = 71^{\circ}$ (correct to the nearest degree)

41. C

Join OC and OB.

$\angle PBO = \angle BPQ = 12^\circ$ (base $\angle$s, isos. Δ)

$\angle BOQ = 2\angle BPQ$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$= 2 \times 12^\circ$

$= 24^\circ$

$\angle COP = \angle PBO$ ($\angle$s in alt. segment)

$= 12^\circ$

$\angle COB + \angle COP + \angle BOQ = 180^\circ$ (adj. $\angle$s on st. line)

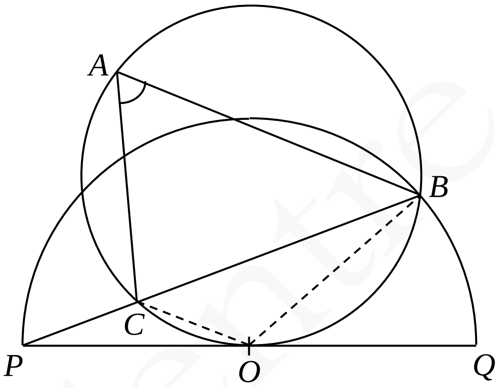
$\angle COB + 12^\circ + 24^\circ = 180^\circ$

$\angle COB = 144^\circ$

$\angle BAC + \angle COB = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle BAC + 144^\circ = 180^\circ$

$\angle BAC = 36^\circ$



42. B

$$\begin{cases} x - y + k = 0 \text{ i.e. } x = y - k \ldots (1) \\ x^2 + y^2 + 2x - 4y - 13 = 0 \ldots (2) \end{cases}$$

Substitute (1) into (2),

$(y - k)^2 + y^2 + 2(y - k) - 4y - 13 = 0$

$y^2 - 2ky + k^2 + y^2 + 2y - 2k - 4y - 13 = 0$

$2y^2 - 2(k + 1)y + k^2 - 2k - 13 = 0$

$\Delta > 0$

$[-2(k + 1)]^2 - 4(2)(k^2 - 2k - 13) > 0$

$k^2 - 6k - 27 < 0$

$(k + 3)(k - 9) < 0$

$-3 < k < 9$

43. B

Number of different teams that can be formed

= Total number of teams formed – number of teams with boys only

$= C_5^{20} - C_5^{12}$

= 14 712

44. D

The required probability

= P("1st number is not 9") × P("2nd number is not 9") × P("3rd number is 9")

$= \frac{3}{6} \times \frac{2}{5} \times \frac{3}{4}$

$= \frac{3}{20}$

45. A

$$m_1 = \frac{x_1+x_2+x_3+\cdots+x_{100}}{100}$$

$$m_2 = \frac{x_1+x_2+x_3+\cdots+x_{100}+m_1}{101}$$

$$= \frac{100m_1+m_1}{101}$$

$$= m_1$$

$\therefore$ I is true.

$\because$ the minimum datum $\leq m_1 \leq$ the maximum datum

$\therefore$ $r_1 = r_2$

$\therefore$ II is true.

$\because$ $m_1 = m_2$

$\therefore$ The second group of numbers are less dispersed.

$\therefore$ $v_2 < v_1$

$\therefore$ III is NOT true.

Alternatively

$$v_1 = \frac{(x_1-m_1)^2+(x_2-m_1)^2+(x_3-m_1)^2+\cdots+(x_{100}-m_1)^2}{100}$$

$$v_2 = \frac{(x_1-m_2)^2+(x_2-m_2)^2+(x_3-m_2)^2+\cdots+(x_{100}-m_2)^2+(m_1-m_2)^2}{101}$$

$$= \frac{(x_1-m_1)^2+(x_2-m_1)^2+(x_3-m_1)^2+\cdots+(x_{100}-m_1)^2+(m_1-m_1)^2}{101} \quad [\because\ m_1 = m_2]$$

$$= \frac{(x_1-m_1)^2+(x_2-m_1)^2+(x_3-m_1)^2+\cdots+(x_{100}-m_1)^2}{101}$$

$$< v_1$$

$\therefore$ III is NOT true.