Suggested Solution for 2013 HKDSE Mathematics(core) Multiple Choice Questions

1. B

$(27 \cdot 9^{n+1})^3$

$=(3^3 \cdot 3^{2(n+1)})^3$

$=(3^{3+2n+2})^3$

$=(3^{2n+5})^3$

$= 3^{6n+15}$

2. D

$\frac{y-1}{c}=\frac{y+1}{d}$

$d(y-1) = c(y+1)$

$dy - d = cy + c$

$dy - cy = c + d$

$y(d - c) = c + d$

$y = \frac{c+d}{d-c}$

3. D

$h\ell - k\ell + hm - km - hn + kn$

$= (h-k)\ell + (h-k)m - (h-k)n$

$= (h-k)(\ell + m - n)$

4. C

0.0504545

= 0.050 (correct to 2 significant figures)

= 0.050 (correct to 3 decimal places)

= 0.05045 (correct to 4 significant figures)

= 0.05045 (correct to 5 decimal places)

5. A

$x - \frac{x-1}{2} > 5$ or $1 < x - 11$

$2x - (x-1) > 10$ or $12 < x$

$2x - x + 1 > 10$ or $x > 12$

$x > 9$ or $x > 12$

$\therefore \quad x > 9$

6. C

$(x-k)^2 = 4k^2$

$(x-k)^2 = (2k)^2$

$x-k = 2k$ or $x-k = -2k$

$x = 3k$ or $x = -k$

Alternatively

$(x-k)^2 = 4k^2$

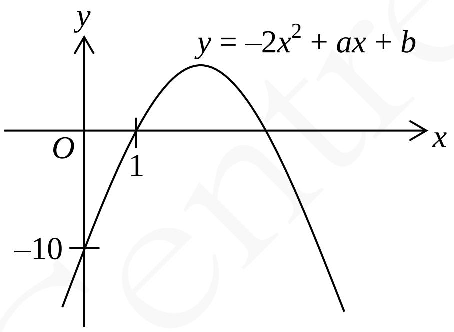
$(x-k)^2 - (2k)^2 = 0$

$[(x-k) + 2k][(x-k) - 2k] = 0$

$(x+k)(x-3k) = 0$

$x = -k$ or $x = 3k$

7. B



Substitute (0, –10) into the function.

$-10 = -2(0)^2 + a(0) + b$

$b = -10$

Substitute (1, 0) into the function.

$0 = -2(1)^2 + a(1) - 10$

$a = 12$

$\therefore \quad y = -2x^2 + 12x - 10$

The equation of the axis of symmetry of the graph is $x = \frac{-12}{2(-2)}$ i.e. $x = 3$.

8. A

$x(x+3a) + a \equiv x^2 + 2(bx + c)$

$x^2 + 3ax + a \equiv x^2 + 2bx + 2c$

$\therefore \quad 3a = 2b$ and $a = 2c$

$\therefore \quad a : b = 2 : 3$ and $a : c = 2 : 1$

i.e. $\quad a : b : c = 2 : 3 : 1$

9. D

By Factor theorem, $f(-1) = 0$.

$(-1)^{13} - 2(-1) + k = 0$

$k = -1$

$\therefore \quad f(x) = x^{13} - 2x - 1$

By Remainder theorem, the required remainder

$= f(1)$

$= (1)^{13} - 2(1) - 1$

$= -2$

10. A

Total cost of the two cars

$= \frac{\$80\ 080}{1+30\%} + \frac{\$80\ 080}{1-30\%}$

$= \$176\ 000$

Total loss $= \$176\ 000 - \$80\ 080 \times 2$

$= \$15\ 840$

11. D

The required interest

$= \$50\ 000(1 + \frac{8\%}{12})^{12} - \$50\ 000$

$= \$4\ 150$

12. C

The actual area = $900 \times 10\ 000$ cm^2

$= 9 \times 10^6$ cm^2

Let the scale of the map be 1 : n. Note that the area of the playground on the map and the actual area of the playground are similar figures.

$\left(\frac{1}{n}\right)^2 = \frac{36}{9\times10^6}$

$n = 500$

13. C

Let $z = \frac{kx}{\sqrt{y}}$ where k is a constant. Then,

$x = k'z\sqrt{y}$ where $k' = \frac{1}{k}$ is a constant.

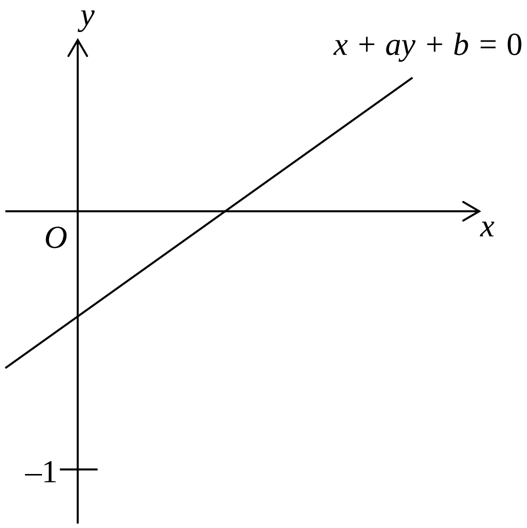
When y is decreased by 64% and z is increased by 25% , the new value of x is given by

$x' = k'[(1 + 25\%)z]\sqrt{(1-64\%)y}$

$= 75\%x$

$\therefore$ x is decreased by 25%.

14. D



Rewrite $x + ay + b = 0$ as $y = -\frac{x}{a} - \frac{b}{a}$.

Slope = $-\frac{1}{a} > 0$ ➔ $a < 0$

$\therefore$ I is true.

Substitute $y = 0$, x-intercept = $-b > 0$ ➔ $b < 0$

$\therefore$ II is true.

y-intercept = $-\frac{b}{a} > -1$

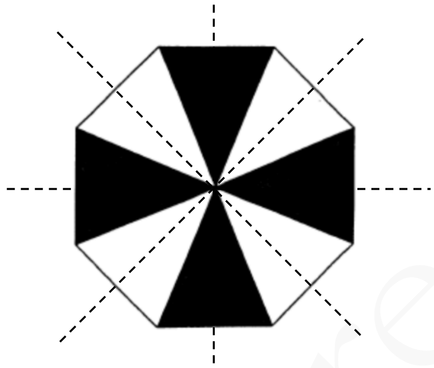
$\frac{b}{a} < 1$

$a < b$ [$\because$ $a < 0$]

$\therefore$ III is true.

15. B

As shown, the number of axes of reflectional symmetry is 4.



16. B

Join BC.

$\angle ACB = 90^{\circ}$ ($\angle$ in semi-circle)

$\sin\angle ABC = \frac{AC}{AB} = \frac{2}{3}$

$\angle ABC \approx 41.8103149^{\circ}$

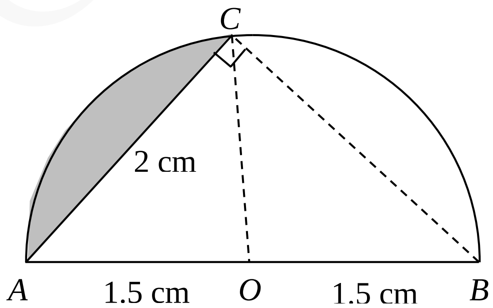
Let O be the centre of the circle.

Note that $OA = OB = OC = 1.5$ cm.

$\therefore\ \angle AOC = 2\angle ABC$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\approx 2 \times 41.8103149^{\circ}$

$\approx 83.62062979^{\circ}$



The area of the shaded region

= Area of sector AOC – area of $\triangle AOC$

$= \pi(1.5)^2 \times \frac{83.62062979^{o}}{360^{o}} - \frac{1}{2} \times 1.5 \times 1.5\sin 83.62062979^{\circ}$

≈ 0.52 cm^2

17. B

Slant height of the right circular cone, ℓ

$=\sqrt{3^2 + 4^2}$

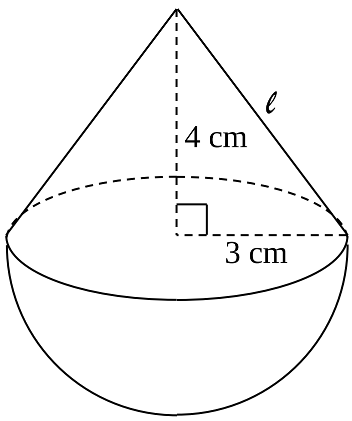
= 5 cm

Total surface area

$= \pi r\ell + 2\pi r^2$

$= \pi(3)(5) + 2\pi(3)^2$

$= 33\pi$ cm^2



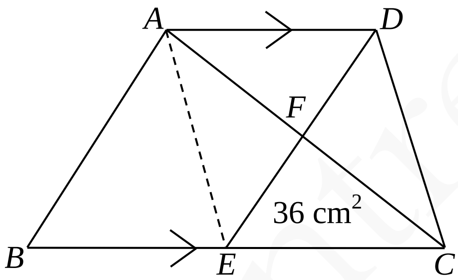
18. C

Note that $AD : BE : EC = 2 : 1.5 : 1.5 = 4 : 3 : 3$

Also note that $\triangle ADF \sim \triangle CEF$.

$\therefore \quad AD : CE = AF : CF = DF : EF = 4 : 3$

Area of $\triangle CDF$: area of $\triangle CEF = DF : EF = 4 : 3$ ($\because$ $\triangle CDF$ and $\triangle CEF$ have the same height.)



Area of $\triangle CDF : 36 = 4 : 3$

Area of $\triangle CDF = 48\text{ cm}^2$

Area of $\triangle CDE$

= Area of CDF + area of $\triangle CEF$

$= 48 + 36$

$= 84\text{ cm}^2$

Join AE.

Area of $\triangle ABE$ = area of $\triangle CDE$ ($\because$ $\triangle ABE$ and $\triangle CDE$ have the same height with $BE = EC$.)

$= 84\text{ cm}^2$

Area of $\triangle ADE$: area of $\triangle CDE = AD : CE = 4 : 3$ ($\because$ $\triangle ADE$ and $\triangle CDE$ have the same height.)

Area of $\triangle ADE : 84 = 4 : 3$

Area of $\triangle ADE = 112\text{ cm}^2$

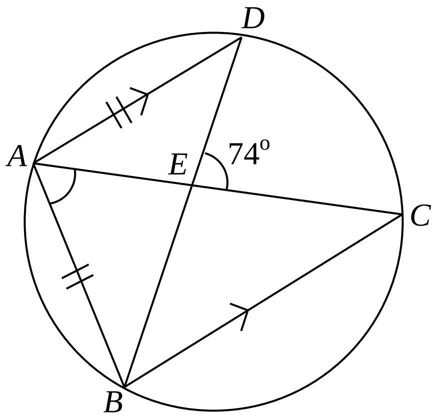
Area of trapezium $ABCD$

= Area of $\triangle ABE$ + area of $\triangle CDE$ + area of $\triangle ADE$

$= 84 + 84 + 112$

$= 280\text{ cm}^2$

19. C



$\angle ACB = \angle ADB$ ($\angle$s in the same segment)

$\angle ADB = \angle DBC$ (alt. $\angle$s, $AD//BC$)

$\angle DBC + \angle ACB = \angle CED = 74^\circ$ (ext. $\angle$ of $\triangle$)

$2\angle DBC = 74^\circ$

$\angle DBC = 37^\circ$

$\therefore \quad \angle ADB = \angle ACB = \angle DBC = 37^\circ$

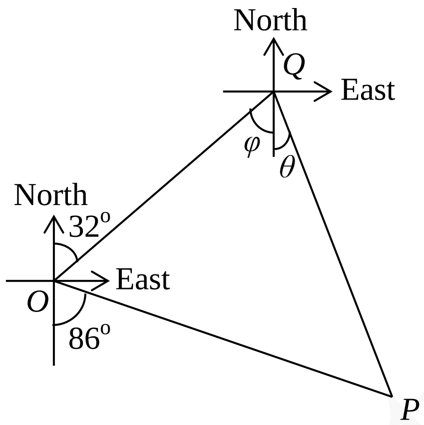
$\angle ABD = \angle ADB = 37^\circ$ (base $\angle$, isos. $\triangle$)

$\angle BAE + \angle ABD + \angle DBC + \angle ACB = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle BAE + 37^\circ + 37^\circ + 37^\circ = 180^\circ$

$\angle BAE = 69^\circ$

20. D



$\angle POQ + 32^o + 86^o = 180^o$ (adj. $\angle$s on st. line)

$\angle POQ = 62^o$

$\angle OPQ = \angle OQP$ (base $\angle$, isos. $\triangle$)

$\angle OPQ + \angle OQP + \angle POQ = 180^o$ ($\angle$ sum of $\triangle$)

$2\angle OQP + 62^o = 180^o$

$\angle OQP = 59^o$

$\varphi = 32^o$ (alt. $\angle$s, // lines)

$\theta + \varphi = 59^o$

$\theta + 32^o = 59^o$

$\theta = 27^o$

$\therefore$ The bearing of P from Q is S27^{o}E.

21. C

Let x be an exterior angle of the regular polygon. Then, an interior angle is $4x$.

$x + 4x = 180^o$

$x = 36^o$

$n = \dfrac{360^o}{36^o} = 10$ (sum of ext. $\angle$ of polygon)

$\therefore$ I is true.

Number of diagonals

$= \dfrac{10(10-3)}{2}$

$= 35$

$\therefore$ II is NOT true.

$\because$ $n = 10$

$\therefore$ The number of folds of rotational symmetry of the polygon is 10.

$\therefore$ III is true.

22. A

Note that $8^2 + 15^2 = 17^2$.

$\triangle ABC$ is a right-angled $\triangle$ with $\angle B = 90^o$.

$\cos A = \dfrac{AB}{AC} = \dfrac{8}{17}$ and $\cos C = \dfrac{BC}{AC} = \dfrac{15}{17}$

$\therefore$ $\cos A : \cos C = \dfrac{8}{17} : \dfrac{15}{17} = 8 : 15$

<u>Alternatively</u>

Let $AB = 8k$, $BC = 15k$ and $AC = 17k$ where k is a constant. By cosine formula,

$\cos A = \dfrac{AB^2+AC^2-BC^2}{2(AB)(AC)} = \dfrac{(8k)^2+(17k)-(15k)^2}{2(8k)(17k)} = \dfrac{8}{17}$

$\cos C = \dfrac{AC^2+BC^2-AB^2}{2(AC)(BC)} = \dfrac{(17k)^2+(15k)-(8k)^2}{2(17k)(15k)} = \dfrac{15}{17}$

$\therefore$ $\cos A : \cos C = \dfrac{8}{17} : \dfrac{15}{17} = 8 : 15$

23. B

$\tan x \tan(90^\circ - x)$

$= \tan x\left(\dfrac{1}{\tan x}\right)$

$= 1$

$\therefore$ I must be true.

$\sin x - \sin(90^\circ - x)$

$= \sin x - \cos x > 0$ if $45^\circ < x < 90^\circ$.

$\therefore$ II may NOT be true.

$\cos x + \cos(90^\circ - x)$

$= \cos x + \sin x > 0$ [$\because$ $\cos x > 0$ and $\sin x > 0$ for $0^\circ < x < 90^\circ$.]

$\therefore$ III must be true.

24. A

Let the coordinates of P be (x, y).

$\sqrt{(x-2)^2 + (y-5)^2} = \sqrt{(x-4)^2 + [y-(-1)]^2}$

$x^2 - 4x + 4 + y^2 - 10y + 25 = x^2 - 8x + 16 + y^2 + 2y + 1$

i.e. $x - 3y + 3 = 0$

25. D

Rewrite the equation of C as $x^2 + y^2 - 2x + 4y - \frac{5}{2} = 0$.

Centre $= (-\frac{-2}{2}, -\frac{4}{2})$ i.e. $(1, -2)$

The radius of C, $r = \sqrt{1^2 + (-2)^2 - \left(-\frac{5}{2}\right)} = \dfrac{\sqrt{30}}{2}$

$\therefore$ I is NOT true.

The mid-point of $PQ = \left(\dfrac{-1+4}{2}, \dfrac{2+0}{2}\right) = \left(\dfrac{3}{2}, 1\right)$

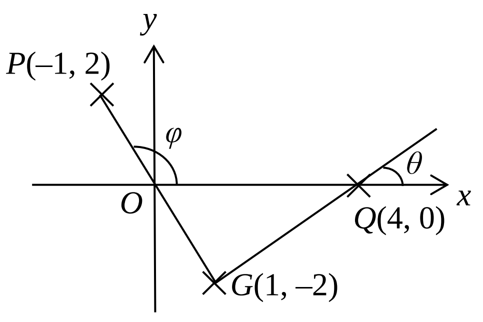
The distance between the mid-point of PQ and the centre of C

$= \sqrt{\left(\frac{3}{2} - 1\right)^2 + [1-(-2)]^2}$

$= \dfrac{\sqrt{37}}{2} > r$

$\therefore$ II is true.

Refer to the figure on the right.



$\tan\theta = \dfrac{-2-0}{1-4} = \dfrac{2}{3}$ and $\tan\varphi = \dfrac{-2-2}{1-(-1)} = -2$

$\theta \approx 33.69006753^\circ$ and $\varphi \approx 116.5650512^\circ$

$\angle PGQ = \varphi - \theta \approx 82.87498365^\circ$ (ext. $\angle$ of $\triangle$)

$\therefore$ III is true.

26. A

From the table shown, the required probability

$= \frac{12}{42}$

$= \frac{2}{7}$

1st number

2nd number	1	2	3	4	5	6	7
1	×	2	3	4	5	6	7
2	2	×	6	8	10	12	14
3	3	6	×	12	15	18	21
4	4	8	12	×	20	24	28
5	5	10	15	20	×	30	35
6	6	12	18	24	30	×	42
7	7	14	21	28	35	42	×

<u>Alternatively</u>

If the two numbers drawn are both odd, then their product is an odd number. Note that there are 4 odd numbers.

The required probability

$= \frac{C_2^4}{C_2^7}$

$= \frac{2}{7}$

27. B

$\because$ The mode = 14

$\therefore$ Two of x, y and z must be 14. Let $x = y = 14$.

$\because$ The mean = 8

$\therefore$ $\frac{14\times3+6+4+5+7+5+z}{9} = 8$

$z = 3$

Rearrange the number in ascending order : 3, 4, 5, 5, 6, 7, 14, 14, 14

$\because$ The median = 6

28. A

The points on the scatter diagram show that y increases when x increases.

29. D

From the distribution, the minimum = 40, the lower quartile, Q_1 = 44, the median = 52, the upper quartile, Q_3 = 65 and the maximum = 95. Note that the median lies near the minimum while Q_3 sits almost at the middle of the distribution. So, D best represents the distribution.

30. B

$\theta = 360^\circ - 60^\circ - 162^\circ - 36^\circ - 68^\circ = 34^\circ$

Proportion representing profit from the sales of pens and notebooks = $60^\circ + 34^\circ = 94^\circ$.

Proportion representing profit from the sales of rulers and pencils = $68^\circ + 36^\circ = 104^\circ > 94^\circ$.

$\therefore$ B is true.

31. B

$a^2 + 4a + 4 = (a + 2)^2$

$a^2 - 4 = (a + 2)(a - 2)$

$a^3 + 8 = (a + 2)(a^2 - 2a + 4)$

$\therefore$ The L.C.M. = $(a - 2)(a + 2)^2(a^2 - 2a + 4)$

32. B

$y = ab^x$ passes through (0, 3).

$3 = ab^0 \Rightarrow a = 3$

As y decreases when x increases, $0 < b < 1$.

$\log_7 y = \log_7(3b^x)$

$= \log_7 b^x + \log_7 3$

$= (\log_7 b)x + \log_7 3$

Slope = $\log_7 b < 0$ [$\because$ $0 < b < 1$]

$\log_7 y$-intercept = $\log_7 3 > 0$

$\therefore$ B is the required graph.

33. A

$A00000E00011_{16}$

$= 10 \times 16^{11} + 14 \times 16^5 + 1 \times 16^1 + 1$

$= 10 \times 16^{11} + 14 \times 16^5 + 17$

16^{11}	16^{10}	16^9	16^8	16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0
A	0	0	0	0	0	E	0	0	0	1	1

34. D

$$\begin{cases} x - \log y = 2 \quad \text{i.e. } x = \log y + 2 \ \ldots (1) \\ x^2 - \log y^2 - 10 = 2 \quad \text{i.e. } x^2 - \log y^2 - 12 = 0 \ \ldots (2) \end{cases}$$

Substitute (1) into (2),

$(\log y + 2)^2 - \log y^2 - 12 = 0$

$(\log y)^2 + 4\log y + 4 - 2\log y - 12 = 0$

$(\log y)^2 + 2\log y - 8 = 0$

$(\log y + 4)(\log y - 2) = 0$

$\log y = -4$ or 2

$y = 10^{-4}$ or 10^2 i.e. $y = \frac{1}{10000}$ or 100

35. D

α and β are the roots of the quadratic equation $x^2 - 3x - 5 = 0$.

The product of roots, $\alpha\beta = -5$

36. A

$i + 2i^2 + 3i^3 + 4i^4$

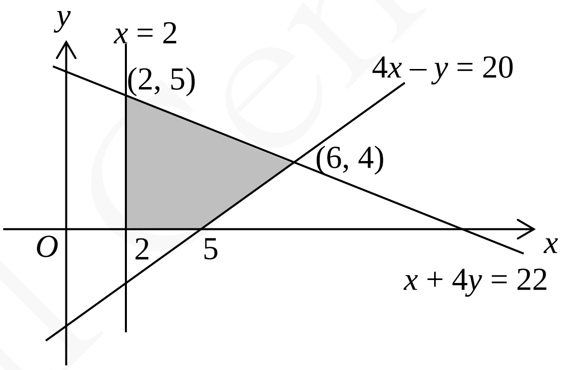
$= i + 2(-1) + 3(-i) + 4(1)$

$= 2 - 2i$

$\therefore$ The real part = 2

37. C

Refer to the figure on the right.



Substitute $x = 2$ into $x + 4y = 22$,

$2 + 4y = 22$

$y = 5$

$$\begin{cases} x + 4y = 22 \ \ldots\ (1) \\ 4x - y = 20 \ \ldots\ (2) \end{cases}$$

Solving (1) and (2),

$x = 6$ and $y = 4$

Let $P(x, y) = 3y - 4x + 15$.

$P(2, 5) = 3(5) - 4(2) + 15 = 22$

$P(6, 4) = 3(4) - 4(6) + 15 = 3$

$P(2, 0) = 3(0) - 4(2) + 15 = 7$

$P(5, 0) = 3(0) - 4(5) + 15 = -5$

$\therefore$ The greatest value of $3y - 4x + 15$ is 22.

<u>Alternatively</u>

The value of $3y - 4x + 15$ is the greatest when y is the greatest and x is the smallest. i.e. (2, 5)

$\therefore$ The greatest value of $3y - 4x + 15$

$= 3(5) - 4(2) + 15$

$= 22$

38. C

$2n - 19 = 25$

$n = 22$

$\therefore$ 25 is a term of the sequence.

$\therefore$ I is true.

$2n - 19 < 0$

$n < 9.5$

$\therefore$ II is NOT true.

1st term, $a = 2(1) - 19 = -17$

Sum of the first n term

$= \dfrac{(-17+2n-19)n}{2}$

$= n^2 - 18n$

$\therefore$ III is true.

39. A

When $x = 0$, $y = 2$, then

$2 = h + k \tan 2(0)^{\circ}$

$h = 2 > 0$

$\therefore$ I is true.

For $0 < x < \frac{\pi}{4}$, $\tan 2x^{\circ} > 0$.

Now, $2 + k \tan 2x^{\circ} < 2$ for $0 < x < \alpha$,

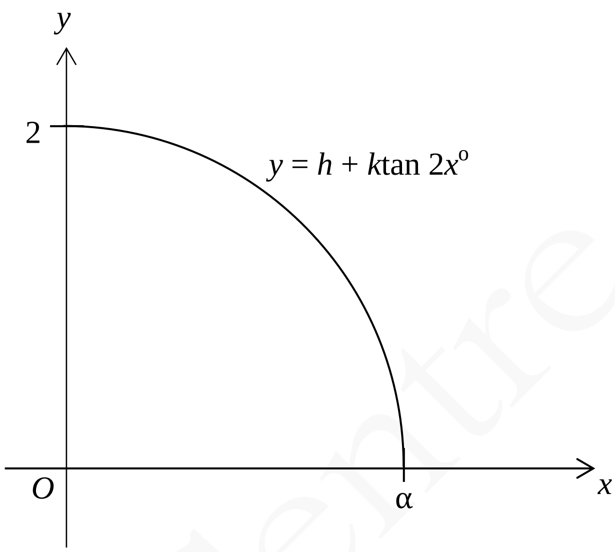
$k \tan 2x^{\circ} < 0$

$k < 0$

$\therefore$ II is true.

$\tan \alpha > 0$ but $\frac{1}{k} < 0$

$\therefore$ III is NOT true.



40. B

Let the length of one side of the tetrahedron be $2L$ cm.

$\frac{L}{d} = \cos 30^{\circ}$

$d = \frac{L}{\cos 30^{o}} = \frac{2L}{\sqrt{3}}$

By Pythagoras' theorem,

$2^2 + \left(\frac{2L}{\sqrt{3}}\right)^2 = (2L)^2$

$L = \frac{\sqrt{6}}{2}$ cm i.e. $2L = \sqrt{6}$ cm

Base area of the tetrahedron

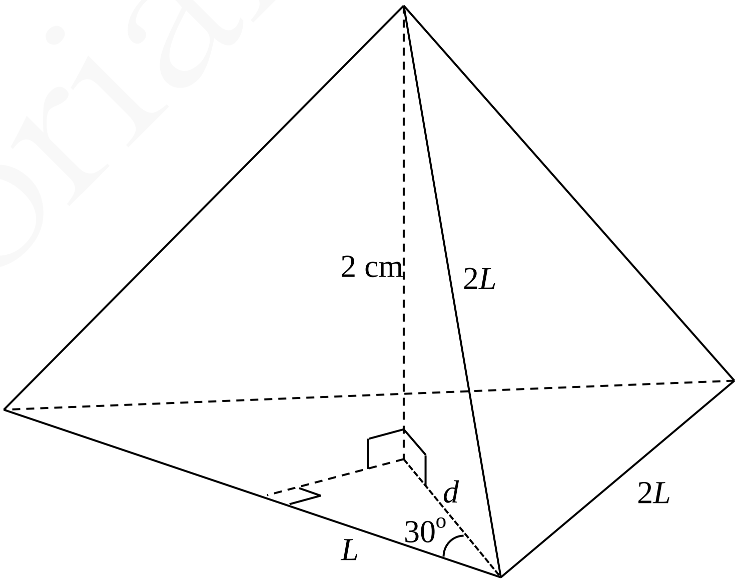
$= \frac{1}{2} \times \sqrt{6} \times \sqrt{6} \sin 60^{\circ}$

$= \frac{3\sqrt{3}}{2}$ cm^2

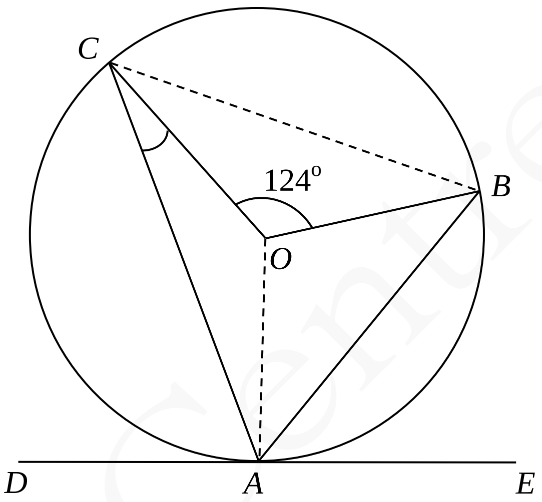
Volume of the tetrahedron

$= \frac{1}{3} \times \frac{3\sqrt{3}}{2} \times 2$

$= \sqrt{3}$ cm^3



41. D



$\angle CAB = \frac{\angle BOC}{2}$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$= \frac{124^o}{2}$

$= 62^o$

$\therefore \quad \angle BAE = \angle CAB = 62^o$

Join OA. (*)

Then, $\angle OAE = \angle OAD = 90^o$ (tangent $\perp$ radius)

$\angle CAO = \angle BAE + \angle CAB - \angle OAE$

$= 62^o + 62^o - 90^o$

$= 34^o$

$\therefore \quad \angle ACO = \angle CAO$ (base $\angle$s, isos. Δ)

$= 34^o$

(*) Alternatively

Join BC.

$\angle ACB = \angle BAE$ ($\angle$s in alt. segment)

$= 62^o$

$\angle OCB = \angle OBC$ (base $\angle$s, isos. Δ)

$\angle OCB + \angle OBC + \angle BOC = 180^o$ ($\angle$ sum of Δ)

$2\angle OCB + 124^o = 180^o$

$\angle OCB = 28^o$

$\therefore \quad \angle ACO = \angle ACB - \angle OCB$

$= 62^o - 28^o$

$= 34^o$

42. B

$$\begin{cases} 3x - 4y + k = 0 \quad \text{i.e. } x = \frac{4y-k}{3} \ \ldots (1) \\ x^2 + y^2 + 2x - 2y - 7 = 0 \ \ldots (2) \end{cases}$$

Substitute (1) into (2),

$(\frac{4y-k}{3})^2 + y^2 + 2(\frac{4y-k}{3}) - 2y - 7 = 0$

$25y^2 + (6 - 8k)y + k^2 - 6k - 63 = 0$

$\Delta \geq 0$

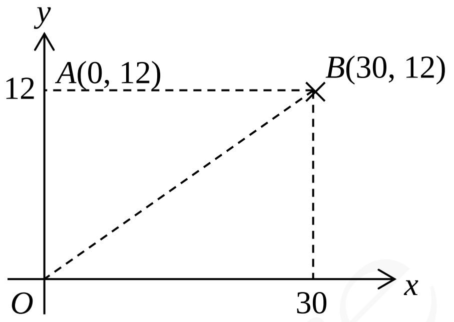
$(6 - 8k)^2 - 4(25)(k^2 - 6k - 63) \geq 0$

$k^2 - 14k - 176 \leq 0$

$(k + 8)(k - 22) \leq 0$

$-8 \leq k \leq 22$

43. A



Note that $\angle OAB = 90^{\circ}$.

Then, OB is a diameter of the circumcircle.

The mid-point of OB is the circumcentre.

$\therefore$ The coordinates of the circumcentre = $\left(\frac{30}{2}, \frac{12}{2}\right)$ i.e. (15, 6)

44. C

Number of phone numbers that can be formed

$= P_3^3 P_5^5$

$= 720$

45. C

Multiplying each number by 3 will change the variance by a factor of $(3)^2$ i.e. 9. Adding 4 to each number has no effect on the variance. Hence, the new variance = 13 × 9 = 117

Alternatively

Let μ be the mean of the numbers x_1, x_2, x_3, x_4 and x_5. Then,

$$\mu = \frac{x_1+x_2+x_3+x_4+x_5}{5}$$

and the variance, $\sigma^2 = \frac{(x_1-\mu)^2+(x_2-\mu)^2+(x_3-\mu)^2+(x_4-\mu)^2+(x_5-\mu)^2}{5} = 13$

The mean of $3x_1 + 4$, $3x_2 + 4$, $3x_3 + 4$, $3x_4 + 4$ and $3x_5 + 4$ is $\frac{3x_1+4+3x_2+4+3x_3+4+3x_4+4+3x_5+4}{5}$

i.e. $3\mu + 4$

The variance of $3x_1 + 4$, $3x_2 + 4$, $3x_3 + 4$, $3x_4 + 4$ and $3x_5 + 4$ is

$$\frac{[3x_1+4-(3u+4)]^2+[3x_2+4-(3u+4)]^2+[3x_3+4-(3u+4)]^2+[3x_4+4-(3u+4)]^2+[3x_5+4-(3u+4)]^2}{5}$$

$= 9\sigma^2$

$= 9 \times 13$

$= 117$