

**Suggested Solution for 2014 HKDSE Mathematics(core) Multiple Choice Questions**

1. B

$$(2n^3)^{-5}$$

$$= \frac{1}{(2n^3)^5}$$

$$= \frac{1}{2^5 n^{3 \times 5}}$$

$$= \frac{1}{32n^{15}}$$

2. A

$$u^2 - v^2 - 5u + 5v$$

$$= (u+v)(u-v) - 5(u-v)$$

$$= (u-v)(u+v-5)$$

3. B

$$\text{L.H.S.} = px(x-1) + x^2$$

$$= px^2 - px + x^2$$

$$= (p+1)x^2 - px$$

$$\text{R.H.S.} = qx(x-2) + 4x$$

$$= qx^2 - 2qx + 4x$$

$$= qx^2 + (4-2q)x$$

$$\begin{cases} p+1 = q \dots (1) \\ -p = 4-2q \dots (2) \end{cases}$$

$$(1) \times 2 + (2),$$

$$p+2 = 4$$

$$p = 2$$

AlternativelySubstitute  $x = 2$  into both sides,

$$p(2)(2-1) + (2)^2 = q(2)(2-2) + 4(2)$$

$$2p + 4 = 8$$

$$p = 2$$

4. B

$$x^2 + ax + a = 1$$

$$x^2 + ax + a - 1 = 0$$

$$\Delta = 0$$

$$a^2 - 4(1)(a-1) = 0$$

$$a^2 - 4a + 4 = 0$$

$$(a-2)^2 = 0$$

$$a = 2$$

5. C

$$\begin{aligned}\because \text{The graph opens upwards.} \\ \therefore m > 0 \\ \because \text{The } y\text{-intercept} < 0 \\ \therefore n < 0\end{aligned}$$

6. D

$$\text{For } 0 > a > b, a^2 < b^2.$$

$\therefore$  I may NOT be true.

$$\text{For } a > b, a + k > b + k$$

$\therefore$  II is true.

$$\therefore \frac{1}{k^2} > 0 \text{ and } a > b$$

$$\therefore \frac{a}{k^2} > \frac{b}{k^2}$$

$\therefore$  III is true.

7. C

$$-3x < 6 \text{ and } 6 < 2x$$

$$x > -2 \text{ and } x > 3$$

$$\therefore x > 3$$

8. A

Let the price of a bowl and that of a cup be \$ $x$  and \$ $y$  respectively.

$$\begin{cases} 2x + 3y = 506 \dots (1) \\ 5x = 4y \text{ i.e. } 5x - 4y = 0 \dots (2) \end{cases}$$

$$(1) \times 4 + (2) \times 3,$$

$$23x = 2024$$

$$x = 88$$

9. A

Let  $x$  be the number of female workers. Then, the number of male workers is  $(1 - 20\%)x$  i.e.  $0.8x$ .

$$x + 0.8x = 792$$

$$1.8x = 792$$

$$x = 440$$

Number of male workers

$$= 0.8(440)$$

$$= 352$$

10. C

Let  $\theta$  and  $r$  be the original angle and radius of the sector respectively.

$$\pi[(1 - 50\%)r]^2 \times \frac{\theta(1-x\%)}{360^\circ} = \pi r^2 \times \frac{\theta}{360^\circ} \times (1 - 90\%)$$

$$0.25(1 - x\%) = 0.1$$

$$x = 60$$

11. A

The lower limit of  $x = (8 - 0.5) \times (10 - 0.5) = 71.25 \text{ cm}^2$

The upper limit of  $x = (8 + 0.5) \times (10 + 0.5) = 89.25 \text{ cm}^2$

$$\therefore 71.25 \leq x < 89.25$$

12. D

Let  $\frac{4}{5a} = \frac{5}{7b} = \frac{7}{9c} = k$  where  $k$  is a constant.

Then,  $a = \frac{4}{5k}$ ,  $b = \frac{5}{7k}$  and  $c = \frac{7}{9k}$ .

$$\therefore \frac{5}{7} < \frac{7}{9} < \frac{4}{5}$$

$$\therefore b < c < a$$

13. C

$z = \frac{ky^3}{x}$  where  $k$  is a constant.

$$k = \frac{xz}{y^3}$$

$$\therefore \frac{y^3}{xz} = \frac{1}{k} \text{ is a constant.}$$

14. D

$$a_4 = a_3 + a_2$$

$$63 = a_3 + 7$$

$$a_3 = 56$$

$$\begin{aligned} a_5 &= a_4 + a_3 \\ &= 63 + 56 \\ &= 119 \end{aligned}$$



18. A

$$\angle ADB = 90^\circ - \theta$$

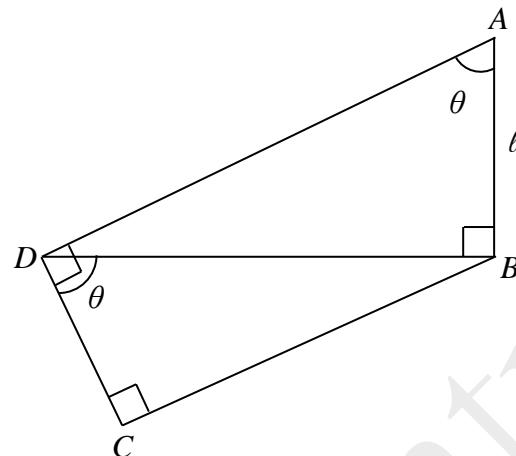
$$\angle CDB = 90^\circ - (90^\circ - \theta) = \theta$$

$$\frac{DB}{AB} = \tan \theta$$

$$DB = AB \tan \theta = \ell \tan \theta$$

$$\frac{CD}{DB} = \cos \theta$$

$$\begin{aligned} CD &= DB \cos \theta \\ &= \ell \tan \theta \cos \theta \\ &= \ell \sin \theta \end{aligned}$$



19. A

$$\begin{aligned} &(\cos(90^\circ + \theta) + 1)(\sin(360^\circ - \theta) - 1) \\ &= (-\sin \theta + 1)(-\sin \theta - 1) \\ &= (-\sin \theta)^2 - 1 \\ &= \sin^2 \theta - 1 \\ &= -\cos^2 \theta \end{aligned}$$

20. B

Join AB.

$$\begin{aligned} \angle ABE &= \angle ADE \quad (\angle \text{s in the same segment}) \\ &= 28^\circ \end{aligned}$$

$$\angle ABC = 90^\circ \quad (\angle \text{ in semi-circle})$$

$$\begin{aligned} \angle CBE &= \angle ABC - \angle ABE \\ &= 90^\circ - 28^\circ \\ &= 62^\circ \end{aligned}$$

Alternatively

Join CD.

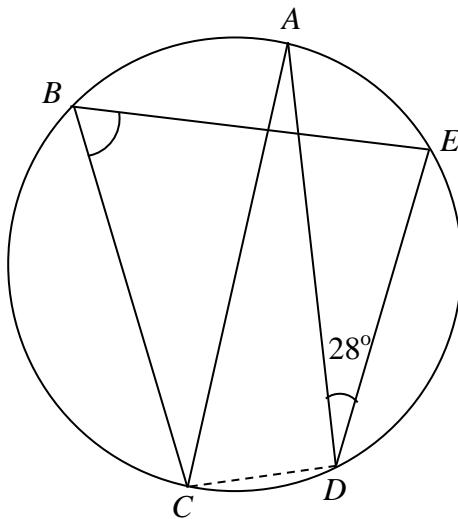
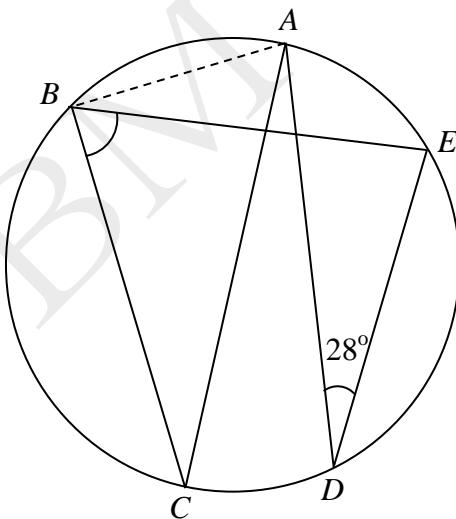
$$\angle ADC = 90^\circ \quad (\angle \text{ in semi-circle})$$

$$\begin{aligned} \angle CDE &= \angle ADC + \angle ADE \\ &= 90^\circ + 28^\circ \\ &= 118^\circ \end{aligned}$$

$$\angle CBE + \angle CDE = 180^\circ \quad (\text{opp. } \angle \text{s, cyclic quad.})$$

$$\angle CBE + 118^\circ = 180^\circ$$

$$\angle CBE = 62^\circ$$



## 21. A

Let  $X$ ,  $Y$  and  $Z$  be the feet of the perpendiculars from  $O$  to  $AB$ ,  $CD$  and  $EF$  respectively.

Then,  $OX = OY = OZ$ . (equal chords, equidistant from centre)

Note that  $\triangle OXQ \cong \triangle OYQ$  and  $\triangle OYR \cong \triangle OZR$ . (RHS)

Let  $\angle ORY = \angle ORZ = x$  and  $\angle OQX = \angle OQY = y$ .

$$\angle PRQ + \angle PQR + \angle QPR = 180^\circ \text{ } (\angle \text{ sum of } \triangle)$$

$$2x + 2y + 38^\circ = 180^\circ$$

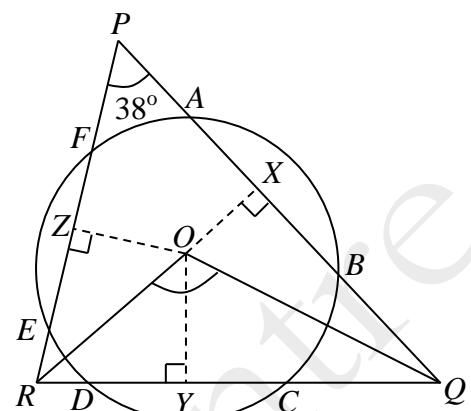
$$x + y = 71^\circ$$

$$\angle QOR + \angle ORQ + \angle OQR = 180^\circ \text{ } (\angle \text{ sum of } \triangle)$$

$$\angle QOR + x + y = 180^\circ$$

$$\angle QOR + 71^\circ = 180^\circ$$

$$\angle QOR = 109^\circ$$



## 22. C

Let  $x$  be an exterior angle of the regular polygon. Then, an interior angle is  $180^\circ - x$ .

$$(180^\circ - x) - x = 100^\circ$$

$$x = 40^\circ$$

$$n = \frac{360}{40} = 9$$

$\therefore$  I is NOT true and II is true.

$$\therefore n = 9$$

$\therefore$  The number of axes of reflectional symmetry of the polygon is 9.

$\therefore$  III is true.

## 23. B

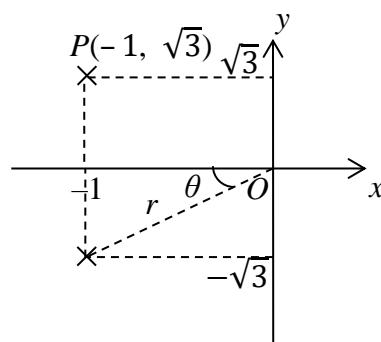
After reflection, the coordinates of the image of  $P$  are  $(-1, -\sqrt{3})$ .

$$\begin{aligned} r &= \sqrt{(-1)^2 + (-\sqrt{3})^2} \\ &= 2 \end{aligned}$$

$$\tan \theta = \frac{\sqrt{3}}{1}$$

$$\theta = 60^\circ$$

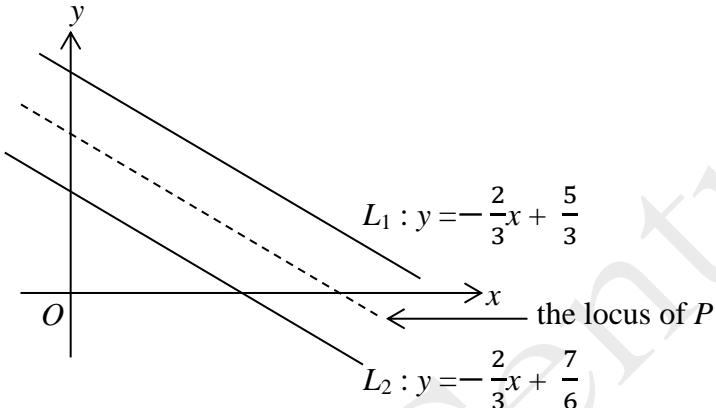
$\therefore$  The polar coordinates of the image are  $(2, 180^\circ + 60^\circ)$ . i.e.  $(2, 240^\circ)$



24. D

Rewrite  $L_1 : y = -\frac{2}{3}x + \frac{5}{3}$  and  $L_2 : y = -\frac{2}{3}x + \frac{7}{6}$ . Note that  $L_1 \parallel L_2$ .

As shown below, the locus of  $P$  is a straight line parallel to both  $L_1$  and  $L_2$  and is equidistant from  $L_1$  and  $L_2$ .



25. D

Rewrite the equations of the straight lines as  $y = -\frac{a}{b}x + \frac{1}{b}$  and  $y = -\frac{c}{d}x + \frac{1}{d}$ .

From the figure,  $\frac{1}{b} > 0$  and  $-\frac{a}{b} > 0$ .

$\therefore b > 0$  and  $a < 0$ .

$\therefore$  I is true.

From the figure,  $\frac{1}{d} > 0$  and  $-\frac{c}{d} < 0$ .

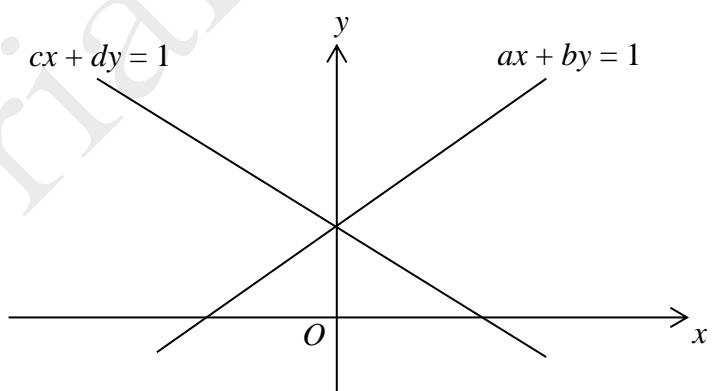
$\therefore d > 0$  and  $c > 0$ .

$\therefore$  II is true.

$$\frac{1}{b} = \frac{1}{d}$$

$\therefore b = d$

$\therefore$  III is true.



26. A

$$\text{Centre} = \left(-\frac{-8}{2}, -\frac{k}{2}\right) \quad \text{i.e. } (4, -\frac{k}{2})$$

$$\text{Then, } \frac{\frac{k}{2} - (-5)}{4 - 6} = -4$$

$$k = -6$$

27. B

$$\frac{m}{m+20} = \frac{1}{m}$$

$$m^2 - m - 20 = 0$$

$$(m - 5)(m + 4) = 0$$

$m = 5$  or  $-4$  (rejected)

28. D

Let  $m$  be the mean height of the teachers.

$$\frac{25m + 145 \times 140}{25 + 140} = 150 \text{ cm}$$

$$m = 178 \text{ cm}$$

29. C

Let  $x$  be his expenditure on transportation that week.

$$x : \$240 = (360^\circ - 160^\circ - 50^\circ - 90^\circ) : 160^\circ$$

$$x : \$240 = 60^\circ : 160^\circ$$

$$x = \$90$$

30. B

$$\therefore 0 \leq h \leq 4$$

$\therefore$  I is NOT true.

Note that  $0 \leq k \leq 9 \dots (1)$

$\therefore$  The range  $\geq 33$

$$\therefore 40 + k - (10 + h) \geq 33$$

$$k - h \geq 3 \text{ i.e. } k \geq 3 + h \dots (2)$$

Combining (1) and (2),  $3 \leq k \leq 9$

$\therefore$  II must be true.

The largest possible value of  $k - h = 9 - 0 = 9$

$$\therefore 3 \leq k - h \leq 9$$

$\therefore$  III is NOT true.

31. A

The H.C.F. of 3, 4 and 6 is 1.

The H.C.F. of  $x^4$ ,  $x$  and  $x^2$  is  $x$ .

The H.C.F. of  $y^2$ ,  $y^5$  and  $y^3$  is  $y^2$ .

$\therefore$  The required H.C.F. is  $xy^2$ .

32. C

For the same value of  $x > 0$ ,

$$b^x > c^x$$

$$\therefore b > c$$

$\therefore$  I is NOT true.

Note that  $b > 1$  and  $c > 1$ .

$$\therefore bc > 1$$

$\therefore$  II is true.

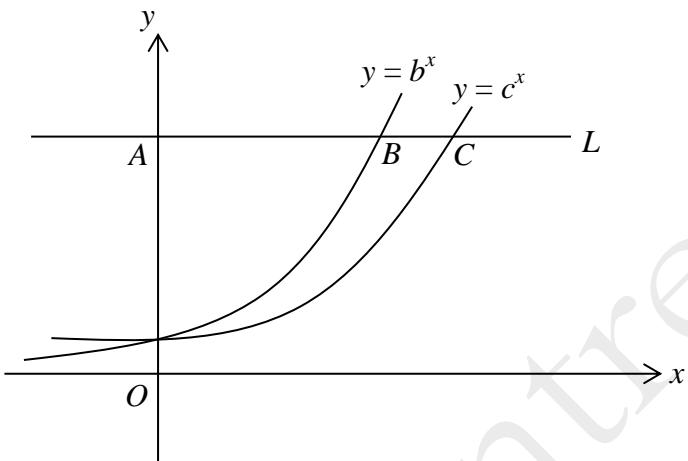
$$b^{AB} = c^{AC}$$

$$\log b^{AB} = \log c^{AC}$$

$$(AB)\log b = (AC)\log c$$

$$\frac{AB}{AC} = \frac{\log c}{\log b} = \log_b c$$

$\therefore$  III is true.



33. B

$$\log 124^{241} = 241 \log 124 \approx 505$$

$$\log 241^{214} = 214 \log 241 \approx 510$$

$$\log 412^{142} = 142 \log 412 \approx 371$$

$$\log 421^{124} = 124 \log 421 \approx 325$$

$\therefore 241^{214}$  is the greatest.

34. C

$$\begin{aligned} & 7 \times 2^{10} + 2^8 + 5 \times 2^3 - 2^3 \\ &= (2^2 + 2 + 1) \times 2^{10} + 2^8 + 4 \times 2^3 \\ &= 2^{12} + 2^{11} + 2^{10} + 2^8 + 2^2 \times 2^3 \\ &= 2^{12} + 2^{11} + 2^{10} + 2^8 + 2^5 \\ &= 1110100100000_2 \end{aligned}$$

$2^{12}$	$2^{11}$	$2^{10}$	$2^9$	$2^8$	$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
1	1	1	0	1	0	0	1	0	0	0	0	0

35. D

Alternatively

$$f(x) = 3x^2 - 6x + k$$

$$= 3(x^2 - 2x + 1) + k - 3$$

$$= 3(x-1)^2 + k - 3$$

$$\therefore k-3 = 7$$

$$k = 10$$

The axis of symmetry is  $x = -\frac{(-6)}{2(3)}$  i.e.  $x = 1$

$$\text{Then, } f(1) = 7.$$

$$3(1)^2 - 6(1) + k = 7$$

$$k = 10$$

36. A

$$\begin{aligned}
 & \frac{\beta^2+4}{\beta+2i} \\
 &= \frac{\beta^2+4}{\beta+2i} \times \frac{\beta-2i}{\beta-2i} \\
 &= \frac{(\beta^2+4)(\beta-2i)}{\beta^2-(2i)^2} \\
 &= \frac{(\beta^2+4)(\beta-2i)}{\beta^2+4} \\
 &= \beta - 2i
 \end{aligned}$$

37. B

$$\frac{2^{2m}}{2^m} = \frac{2^{3m}}{2^{2m}} = \frac{2^{4m}}{2^{3m}} = 2^m$$

$\therefore$  I is a geometric sequence.

$$\frac{2m^2}{m} = 2m \text{ but } \frac{3m^4}{2m^2} = \frac{3m^2}{2}$$

$\therefore$  II is NOT a geometric sequence.

$$\frac{\log m^2}{\log m} = \frac{\log m^4}{\log m^2} = \frac{\log m^8}{\log m^4} = 2$$

$\therefore$  III is a geometric sequence.

38. A

$y = 1 - f(x)$  implies a reflection about the  $x$ -axis and a translation of 1 unit upwards.

Note that:-

B shows a reflection about the  $x$ -axis and a translation of 1 unit to the left.

C shows a reflection about the  $x$ -axis and a translation of 1 unit downwards.

D shows a reflection about the  $x$ -axis and a translation of 1 unit to the right.

39. D

$$7\sin^2 x = \sin x$$

$$7\sin^2 x - \sin x = 0$$

$$\sin x(7\sin x - 1) = 0$$

$$\sin x = 0 \text{ or } \sin x = \frac{1}{7}$$

$$x = 0^\circ, 8.21^\circ, 172^\circ, 180^\circ \text{ or } 360^\circ$$

40. D

Let  $E$  be the foot of the perpendicular from  $A$  to  $CD$ . Then,  $\theta = \angle AEB$ .

By Pythagoras' theorem,

$$\begin{aligned} CD^2 &= BC^2 + BD^2 \\ &= 8^2 + 15^2 \end{aligned}$$

$$CD = 17 \text{ m}$$

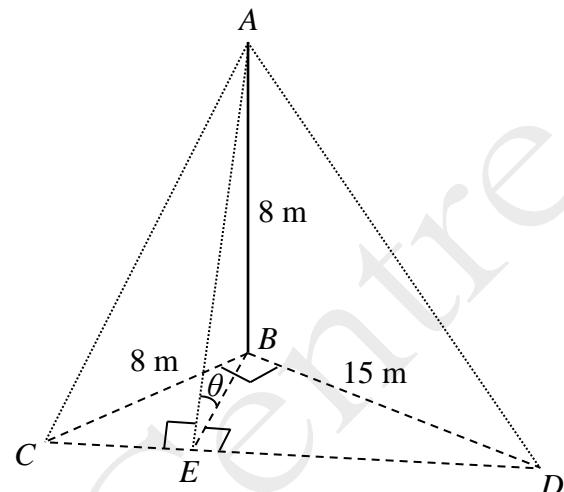
$$\text{Area of } \triangle BCD = \frac{1}{2} \times BC \times BD = \frac{1}{2} \times BE \times CD$$

$$\therefore \frac{1}{2} \times 8 \times 15 = \frac{1}{2} \times BE \times 17$$

$$BE = \frac{120}{17} \text{ m}$$

$$\tan \theta = \frac{AB}{BE} = \frac{8}{\frac{120}{17}}$$

$$= \frac{17}{15}$$



41. C

Note that  $\angle IRS = \angle IRQ = 12^\circ$  ( $\because I$  is the in-centre of  $\triangle QRS$ )

$$\angle SRQ = \angle IRS + \angle IRQ$$

$$= 12^\circ + 12^\circ$$

$$= 24^\circ$$

$$\angle QSR = \angle QPS \text{ ( } \angle \text{s in alt. segment)}$$

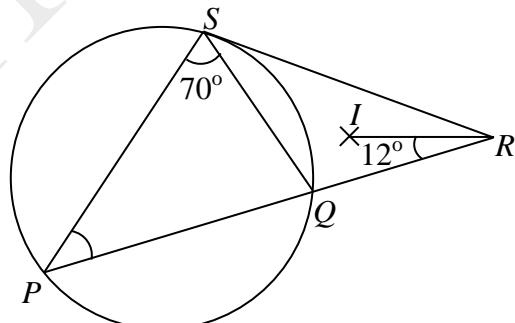
$$\angle PSR = \angle PSQ + \angle QSR$$

$$= 70^\circ + \angle QPS$$

$$\angle QPS + \angle PSR + \angle SRQ = 180^\circ \text{ ( } \angle \text{ sum of } \Delta)$$

$$\angle QPS + (70^\circ + \angle QPS) + 24^\circ = 180^\circ$$

$$\angle QPS = 43^\circ$$



42. C

$$\begin{cases} x - y = k \text{ i.e. } y = x - k \dots (1) \\ x^2 + y^2 + 2x - 4y - 1 = 0 \dots (2) \end{cases}$$

Substitute (1) into (2),

$$x^2 + (x - k)^2 + 2x - 4(x - k) - 1 = 0$$

$$x^2 + x^2 - 2kx + k^2 + 2x - 4x + 4k - 1 = 0$$

$$2x^2 - 2(1+k)x + k^2 + 4k - 1 = 0$$

The  $x$ -coordinate of the mid-point of  $AB$

$$= \frac{1}{2} \cdot \frac{2(1+k)}{2}$$

$$= \frac{1+k}{2}$$

43. B

Number of teams that can be formed

$$= C_2^{13} C_3^{17}$$

$$= 53\,040$$

44. D

Let  $\mu$  and  $\sigma$  be the mean and the standard deviation of the examination scores respectively.

$$\frac{55-\mu}{\sigma} = -3 \quad \text{i.e. } \mu - 3\sigma = 55 \dots (1)$$

$$\frac{95-\mu}{\sigma} = 2 \quad \text{i.e. } \mu + 2\sigma = 95 \dots (2)$$

$$(1) \times 2 + (2) \times 3,$$

$$5\mu = 395$$

$$\mu = 79$$

45. B

Multiplying each number by  $-1$  will change the variance by a factor of  $(-1)^2$  i.e. 1. Adding 14 to each number has no effect on the variance. Hence, the variance remains unchanged.

Alternatively

Let  $\mu$  be the mean of the numbers. Then,

$$\mu = \frac{a+b+c+d}{4}$$

$$\text{and the variance, } \sigma^2 = \frac{(a-\mu)^2 + (b-\mu)^2 + (c-\mu)^2 + (d-\mu)^2}{4}$$

The mean of  $14 - a$ ,  $14 - b$ ,  $14 - c$  and  $14 - d$  is  $\frac{14-a+14-b+14-c+14-d}{4}$  i.e  $14 - \mu$ .

The variance of  $14 - a$ ,  $14 - b$ ,  $14 - c$  and  $14 - d$  is

$$\frac{[14-a-(14-\mu)]^2 + [14-b-(14-\mu)]^2 + [14-c-(14-\mu)]^2 + [14-d-(14-\mu)]^2}{4}$$

$$= \sigma^2$$