Suggested Solution for 2015 HKDSE Mathematics(core) Multiple Choice Questions

1. D

$(x + 1)(x^2 + x + 1)$

$= x(x^2 + x + 1) + 1 \bullet (x^2 + x + 1)$

$= x^3 + x^2 + x + x^2 + x + 1$

$= x^3 + 2x^2 + 2x + 1$

2. D

$\dfrac{(3y^6)^4}{3y^2}$

$= \dfrac{3^4 y^{6\times4}}{3y^2}$

$= 3^{4-1}y^{24-2}$

$= 27y^{22}$

3. A

$\begin{cases} p + 3q = 4 \ldots (1) \\ 5p + 9q = 2 \ldots (2) \end{cases}$

(2) – (1) × 3,

$2p = -10$

$p = -5$

Alternatively

From (1), $p = 4 - 3q \ldots (3)$

Substitute (3) into (2),

$5(4 - 3q) + 9q = 2$

$20 - 15q + 9q = 2$

$6q = 18$

$q = 3$

Then, $p = 4 - 3(3)$

$= -5$

4. D

0.0023456789

= 0.002346 (correct to 6 decimal places)

= 0.00234568 (correct to 6 significant figures)

5. B

R.H.S.

$= (x + 4)(x - m) + 6$

$= x^2 + (4 - m)x + 6 - 4m$

By comparing the coefficients of x,

$m = 4 - m$

$m = 2$

By comparing the constant terms,

$n = 6 - 4(2) = -2$

Alternatively

Substitute $x = 0$,

$n = 6 - 4m \ldots (1)$

Substitute $x = -4$,

$(-4)^2 + m(-4) + n = 6$

$4m = n + 10 \ldots (2)$

Substitute (2) into (1),

$n = 6 - (n + 10) \Rightarrow n = -2$

6. A

$18 + 7x > 4$ or $5 - 2x < 3$

$7x > -14$ or $2x > 2$

$x > -2$ or $x > 1$

$\therefore \quad x > -2$

7. A

$\because \quad \beta$ is a root of the equation $4x^2 - 5x - 1 = 0$

$\therefore \quad 4\beta^2 - 5\beta - 1 = 0$

$8\beta^2 - 10\beta - 2 = 0$

$10\beta - 8\beta^2 = -2$

$7 + 10\beta - 8\beta^2 = 7 - 2 = 5$

8. D

$\because$ The graph opens upwards.

$\therefore \quad a > 0$

$\because$ The axis of symmetry is $x = -b < 0$

$\therefore \quad b > 0$

9. B

Let x be the original price of the souvenir.

New price of the souvenir, $x' = (1 + 70\%)(1 - 60\%)x$

$= 0.68x$

The percentage change $= \dfrac{0.68x - x}{x} \times 100\%$

$= -32\%$

10. D

The required amount

$= \$50\,000 \times (1 + \dfrac{6\%}{4})^{3 \times 4}$

$= \$59\,781$(correct to the nearest dollar)

11. C

$a : c = 5 : 3 = 10 : 6$

$b : c = 3 : 2 = 9 : 6$

$\therefore \quad a : b : c = 10 : 9 : 6$

Let $a = 10k$, $b = 9k$ and $c = 6k$ where k is a constant.

$(a + c) : (b + c)$

$= (10k + 6k) : (9k + 6k)$

$= 16 : 15$

12. D

Let $z = kx^3y^2$ where k is a constant.

$14 = k(2)^3(1)^2$

$k = \frac{7}{4}$

Then, when $x = 3$ and $y = -2$,

$z = \frac{7}{4}(3)^3(-2)^2$

$= 189$

Alternatively

$14 = k(2)^3(1)^2$ … (1)

$z = k(3)^3(-2)^2$ … (2)

(2) ÷ (1),

$\frac{z}{14} = \frac{27}{2}$

$z = 189$

13. B

Let T(n) be the number of dots in the nth pattern. Then,

T(1) = 5

T(2) = 5 + 4 = 9

T(3) = 9 + 4 = 13

T(4) = 13 + 4 = 17

T(5) = 17 + 4 = 21

T(6) = 21 + 4 = 25

14. C

Greatest possible value of n

$< \frac{(5+0.5)\times 1000}{10-0.5}$

$\approx$ 578.9473684

15. C

By Pythagoras' theorem,

$CN^2 + DN^2 = CD^2$

$CN^2 + 6^2 = 10^2$

$CN = 8$ cm

By Pythagoras' theorem,

$AN^2 + EN^2 = AE^2$

$AN^2 + 5^2 = 13^2$

$AN = 12$ cm

$AC = 8 + 12 = 20$ cm

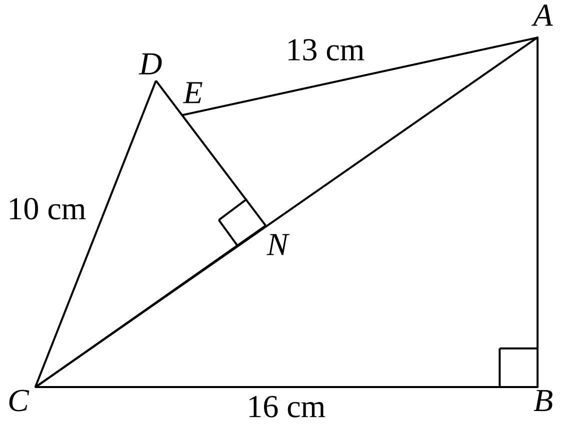
By Pythagoras' theorem,

$AB^2 + BC^2 = AC^2$

$AB^2 + 16^2 = 20^2$

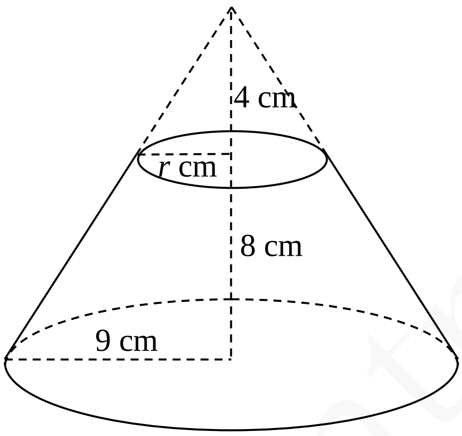
$AB = 12$ cm

Area of $\triangle ABC = \frac{1}{2} \times 12 \times 16 = 96$ cm^2



16. B

Let r cm be the radius of the upper surface.



$\frac{r}{9} = \frac{4}{12}$

$r = 3$

Volume of the frustum

$= \frac{1}{3}\pi(9)^2(12) - \frac{1}{3}\pi(3)^2(4)$

$= 312\pi$ cm^3

Alternatively,

Volume of the circular cone

$= \frac{1}{3}\pi(9)^2(12)$

$= 324\pi$ cm^3

Let V cm^3 be the volume of the upper cone being cut off.

$\frac{V}{324\pi} = \left(\frac{4}{12}\right)^3$ [For similar figures]

$V = 12\pi$ cm^3

Volume of the frustum

$= 324\pi - 12\pi$

$= 312\pi$ cm^3

17. C

Note that $\triangle EDF \sim \triangle ECB$.

$\therefore$ $DF : CB = DE : EC = 2 : 3$

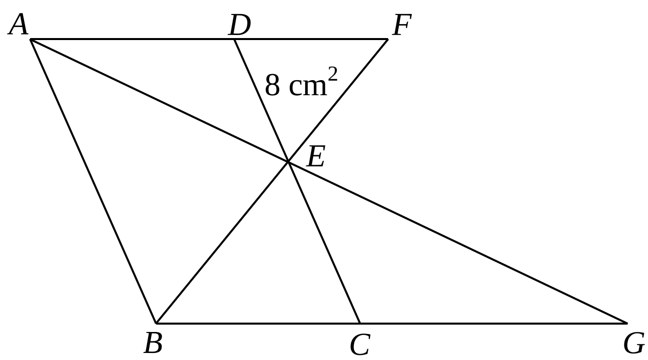
$\because$ $AD = BC$ (property of // gram)

$\therefore$ $DF : AD = 2 : 3$

Area of $\triangle ADE$: area of $\triangle DEF = AD : DF = 3 : 2$ ($\because$ $\triangle ADE$ and $\triangle DEF$ have the same height.)

Area of $\triangle ADE : 8 = 3 : 2$

Area of $\triangle ADE = 12$ cm^2



Note also that $\triangle DEA \sim \triangle CEG$.

$\therefore$ $\frac{\text{Area of } \Delta CEG}{\text{Area of } \Delta DEA} = \left(\frac{CE}{DE}\right)^2$

i.e. $\frac{\text{Area of } \Delta CEG}{12} = \left(\frac{3}{2}\right)^2$

$\therefore$ Area of $\triangle CEG = 27$ cm^2

18. A

$\frac{AC}{AB}$ = tan β

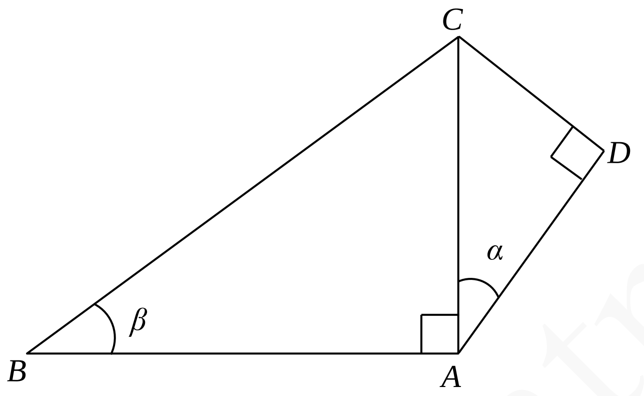
$AC = AB \tan\beta$ … (*)

$\frac{AD}{AC}$ = cos α

$AD = AC \cos\alpha$

$= AB \tan\beta \cos\alpha$ [From (*)]

$\therefore \frac{AD}{AB} = \cos\alpha \tan\beta$



19. C

$$\frac{\cos 180^o}{1+\sin(90^o+\theta)} + \frac{\cos 360^o}{1+\sin(270^o+\theta)}$$

$$= \frac{-1}{1+\cos\theta} + \frac{1}{1-\cos\theta}$$

$$= \frac{-(1-\cos\theta)+1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{-1+\cos\theta+1+\cos\theta}{1-\cos^2\theta}$$

$$= \frac{2\cos\theta}{\sin^2\theta}$$

20. C

Join AC.

$\angle ACD = 90^o$ ($\angle$ in semi-circle)

$\overset{\frown}{BC} = \overset{\frown}{CD}$ (equal chords, equal arcs)

$\angle BAC = \angle CAD$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\therefore \angle CAD = \angle BAC = \frac{58^o}{2} = 29^o$

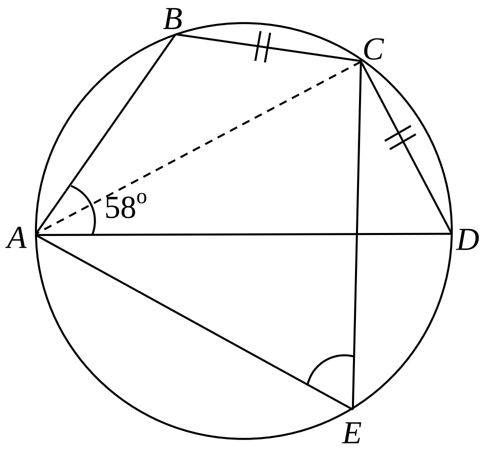
$\angle CDA + \angle CAD + \angle ACD = 180^o$ ($\angle$ sum of $\triangle$)

$\angle CDA + 29^o + 90^o = 180^o$

$\angle CDA = 61^o$

$\angle AEC = \angle CDA$ ($\angle$s in the same segment)

$= 61^o$



21. B

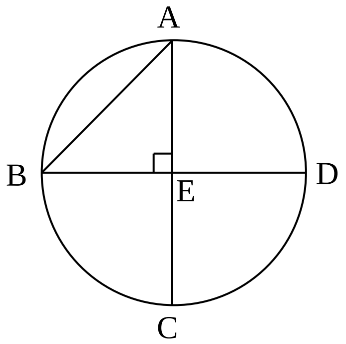
Note that E is the centre of the circle.

Then, the radius = $AE = BE$ = 12 cm.

Area of $\triangle AEB$

$= \frac{1}{2} \times 12 \times 12$

$= 72 \text{ cm}^2$



22. A

Let x be an exterior angle of the regular polygon.

$5x + x = 180^o$

$x = 30^o$

Then, each interior angle $= 5(30^o) = 150^o$

∴ I is true.

Number of sides $= \frac{360^o}{30^o} = 12$ (sum of ext. ∠s of polygon)

Number of diagonals $= \frac{12(12-3)}{2} = 54$

∴ II is NOT true.

Number of folds of rotational symmetry is 12.

∴ III is NOT true.

23. C

After reflection, the coordinates of the image of A are $(-\sqrt{3}, -1)$.

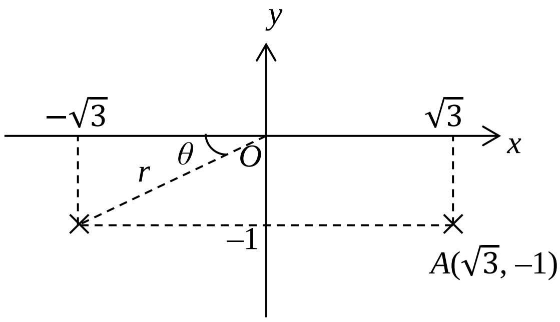
$r = \sqrt{(-\sqrt{3})^2 + (-1)^2}$

$= 2$

$\tan\theta = \frac{1}{\sqrt{3}}$

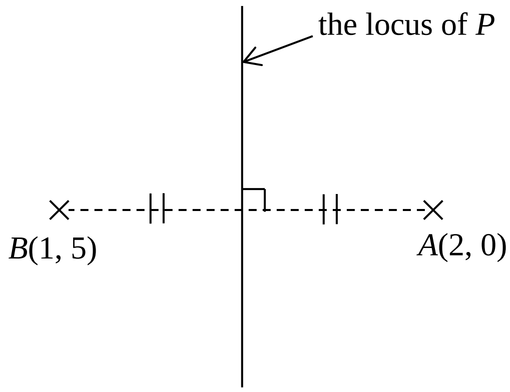
$\theta = 30^o$

∴ The polar coordinates of the image are $(2, 180^o + 30^o)$. i.e. $(2, 210^o)$



24. A

As shown below, the locus of P is the perpendicular bisector of AB.

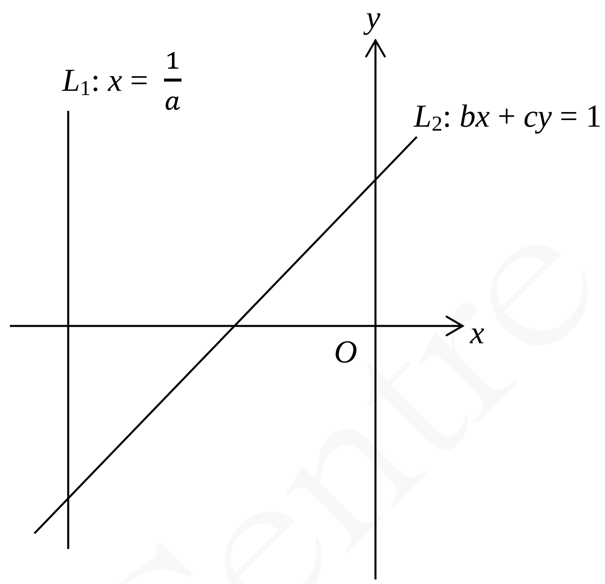


25. B

For L_1, $x = \frac{1}{a} < 0$

$\therefore$ $a < 0$

$\therefore$ I is true.



For L_2, when $y = 0$, $bx = 1$. i.e. $x = \frac{1}{b} < 0$

$\therefore$ $b < 0$

Also, from the figure, $\frac{1}{b} > \frac{1}{a}$.

$\therefore$ $a > b$

$\therefore$ II is NOT true.

For L_2, when $x = 0$, $cy = 1$. i.e. $y = \frac{1}{c} > 0$

$\therefore$ $c > 0$

$\therefore$ III is true.

26. C

Radius = 4

The equation of C is

$[x - (-4)]^2 + (y - 3)^2 = 4^2$

i.e. $x^2 + y^2 + 8x - 6y + 9 = 0$.

27. A

Refer to the table. There are 6 outcomes giving the sum of the numbers as 7.

Expected gain

$= \frac{6}{36} \times \$36 + \frac{30}{36} \times \$6$

$= \$11$

The number on 1st die

The number on 2nd die

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

28. B

The required probability

$= \frac{8}{12+16+8+4+4}$

$= \frac{2}{11}$

29. A

The inter-quartile range

$= 45 - 25$

$= 20$

30. B

Note that $r = 3$.

$\because \quad 3 \le m \le 5$

$\therefore \quad 3 \le m = q \le 5$

$$\frac{2\times2+3\times6+5\times2+6+8\times2+9+10}{15} \le p \le \frac{2\times2+3\times5+5\times3+6+8\times2+9+10}{15}$$

i.e. $\frac{73}{15} \le p \le 5$

When $q = 5$, I is NOT true.

$r(=3) < \frac{73}{15} \le p \le 5$

$\therefore$ II must be true.

When $q = 3$, III is NOT true.

31. C

$\frac{1}{x^2-2x+1} - \frac{1}{x^2+x-2}$

$= \frac{1}{(x-1)^2} - \frac{1}{(x-1)(x+2)}$

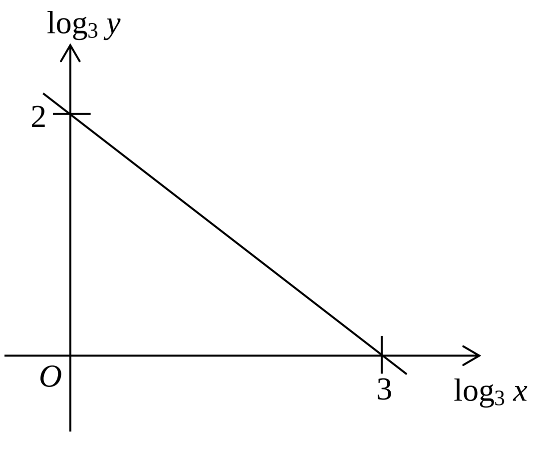
$= \frac{x+2-(x-1)}{(x-1)^2(x+2)}$

$= \frac{3}{(x-1)^2(x+2)}$

32. A

Slope of the graph $= \frac{2-0}{0-3} = -\frac{2}{3}$ and $\log_3 y$–intercept of the graph = 2

Equation of the straight line is



$\log_3 y = -\frac{2}{3}\log_3 x + 2$ [c.f. $y = mx + c$]

$\log_3 y = \log_3 x^{-\frac{2}{3}} + \log_3 3^2$ [using $a\log b = \log b^a$]

$\log_3 y = \log_3 (9x^{-\frac{2}{3}})$ [using $\log a + \log b = \log(ab)$]

$y = 9x^{-\frac{2}{3}}$

$x^{\frac{2}{3}}y = 9$

$(x^{\frac{2}{3}}y)^3 = 9^3$

$x^2y^3 = 729$

33. A

$11 = 2^3 + 2^1 + 2^0$

$= 1011_2$

$11 + 2^6 + 2^{10} + 2^{11}$

$= 110001001011_2$

2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	0	0	0	1	0	0	1	0	1	1

34. B

$\alpha + \beta = -k$ and $\alpha\beta = -2$

$\alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= (-k)^2 - 2(-2)$

$= k^2 + 4$

35. C

$z = (a + 5)i^6 + (a - 3)i^7$

$= (a + 5)i^2 + (a - 3)i^3 \quad [\because \ i^4 = 1]$

$= (a + 5)(-1) + (a - 3)(-i)$

$= -a - 5 - (a - 3)i$

If z is a real number,

$a - 3 = 0$

$a = 3$

36. D

The system of inequalities that define the shaded region is

$$\begin{cases} x \le 4 \ \ldots \ (1) \\ y \le x + 5 \ \ldots \ (2) \\ x + 2y \ge 10 \ \ldots \ (3) \end{cases}$$

From (1), $a \le 4$.

$\therefore$ I is true.

From (2), $b \le a + 5$ i.e. $a \ge b - 5$

$\therefore$ II is true.

From (3), $a + 2b \ge 10$ i.e. $a \ge 10 - 2b$

$\therefore$ III is true.

37. D

Let r be the common ratio of the geometric sequence.

$r^2 = \frac{x_8}{x_6} = \frac{96}{216} = \frac{4}{9}$

$r = \frac{2}{3}$ or $-\frac{2}{3}$

For $r = \frac{2}{3}$,

$x_3 = \frac{x_6}{r^3} = \frac{216}{\left(\frac{2}{3}\right)^3} = 729$

For $r = -\frac{2}{3}$,

$x_3 = \frac{x_6}{r^3} = \frac{216}{\left(-\frac{2}{3}\right)^3} = -729$

∴ I may NOT be true.

$\frac{x_5}{x_7} = \frac{1}{r^2} = \frac{9}{4} > 1$

∴ II is true.

$x_2 = \frac{x_6}{r^4} = \frac{216}{\left(\frac{4}{9}\right)^2} = \frac{2187}{2}$

Note that x_{2n} are positive numbers for $n = 1, 2, 3, \ldots$

$x_2 + x_4 + x_6 + \ldots + x_{2n}$

$< x_2 + x_4 + x_6 + \ldots$

$= \frac{\frac{2187}{2}}{1-\frac{9}{4}}$

$= 1968.3$

< 2015

∴ III is true.

38. B

$\cos^2 x - \sin x = 1$

$(1 - \sin^2 x) - \sin x = 1$

$\sin^2 x + \sin x = 0$

$\sin x(\sin x + 1) = 0$

$\sin x = 0$ or $\sin x = -1$

$x = 0^\circ, 180^\circ$ or 270°

∴ There are 3 roots.

39. D

From the figure,

$\sin[k(75)^{o} + \theta] = 0$ and $\sin[k(165)^{o} + \theta] = 0$.

i.e. $k(75)^{o} + \theta = 180^{o}$ … (1)

and $k(165)^{o} + \theta = 360^{o}$ … (2)

Solving (1) and (2),

$k = 2$ and $\theta = 30^{o}$

Alternatively

From the figure, the period of the graph = 180^{o}.

$\therefore$ It is a reduction of the graph $y = \sin x^{o}$ to 2 times along the x-axis. i.e. $k = 2$

The graph shows a shift of 15^{o} to the left. Then,

$y = \sin[2(x + 15)^{o}]$

$= \sin(2x + 30^{o})$ i.e. $\theta = 30^{o}$

40. C

Let G be the centre of the circle. Then, G is the mid-point of BD and GB, GC and GD are the radii.

$\angle GBA = \angle GCA = 90^{o}$ (tangent $\perp$ radius)

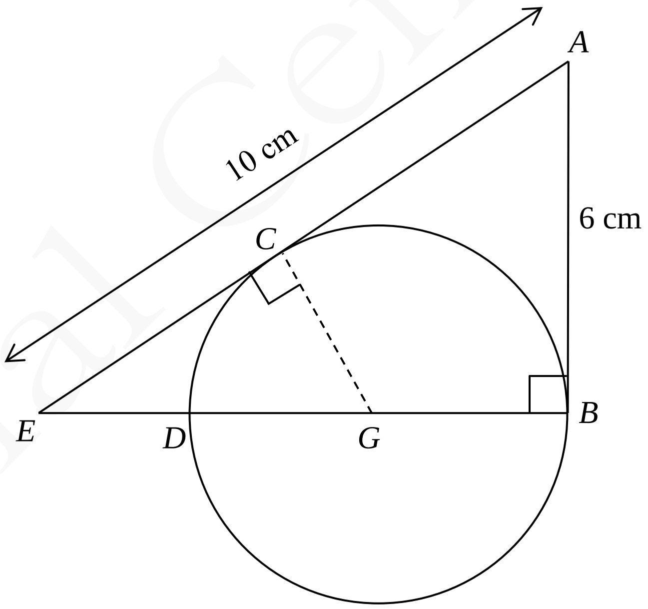
$AC = AB$ (tangent properties)

$= 6$ cm

$CE = AE - AC$

$= 10 - 6$

$= 4$ cm



By Pythagoras' theorem,

$AB^2 + BE^2 = AE^2$

$6^2 + BE^2 = 10^2$

$BE = 8$ cm

Let r cm be the radius of the circle.

Then, $GE = (8 - r)$ cm

By Pythagoras' theorem,

$GC^2 + CE^2 = GE^2$

$r^2 + 4^2 = (8 - r)^2$

$r = 3$ cm

$BD = 2r = 6$ cm

41. B

$$\begin{cases} x + y + 4 = 0 \quad \text{i.e. } x = -y - 4 \ \ldots (1) \\ x^2 + y^2 + 2x - 6y + k = 0 \ \ldots (2) \end{cases}$$

Substitute (1) into (2),

$(-y - 4)^2 + y^2 + 2(-y - 4) - 6y + k = 0$

$y^2 + 8y + 16 + y^2 - 2y - 8 - 6y + k = 0$

$2y^2 + 8 + k = 0$

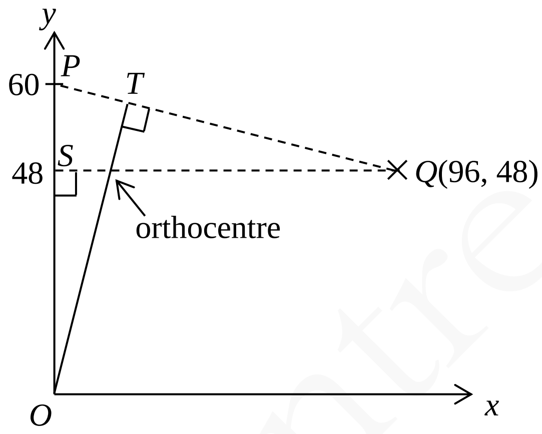
$\Delta = 0$

$\therefore \quad \Delta = 0^2 - 4(2)(8 + k) = 0$

$\therefore \quad k = -8$

42. A

Let T be the foot of the perpendicular from O to PQ.



Slope of OT × slope of PQ = –1

Slope of $OT \times \dfrac{60-48}{0-96} = -1$

Slope of OT = 8

∴ The equation of OT is $y = 8x$.

The orthocentre is the intersection of QS and OT.

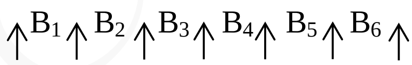
Note that the y-coordinate of the orthocentre is 48.

Then, $48 = 8x$

$x = 6$

43. C

The girls can occupy the indicated positions (marked with ↑).



Number of queues that can be formed

$= P_6^6 \times P_2^7$

= 30 240

<u>Alternatively</u>

Consider the two girls are next to each other. Take them as a group that forms a queue with the 6 boys.

Number of queues that can be formed

$= P_7^7 \times P_2^2$

= 100 80

Number of queues required

$= P_8^8 - 100\ 80$

= 30 240

44. D

The required probability

= P(“choosing bag P”) × P(“choosing a green ball”) + P(“choosing bag Q”) × P(“choosing a green ball”)

$= \dfrac{1}{2} \times \dfrac{4}{6} + \dfrac{1}{2} \times \dfrac{3}{4}$

$= \dfrac{17}{24}$

45. B

$x_1 = \frac{a_1 + a_2 + a_3 + \ldots + a_{49} + a_{50}}{50}$ i.e. $a_1 + a_2 + a_3 + \ldots + a_{49} + a_{50} = 50x_1$ … (1)

$x_2 = \frac{a_1 + a_2 + a_3 + \ldots + a_{48} + a_{49}}{49}$ i.e. $a_1 + a_2 + a_3 + \ldots + a_{48} + a_{49} = 49x_2$ … (2)

Substitute (2) into (1),

$49x_2 + a_{50} = 50x_1$

$49x_2 + x_1 = 50x_1$ [$\because$ $x_1 = a_{50}$]

$x_1 = x_2$

$\therefore$ I is true.

Without loss of generality, let $a_1 \le a_2 \le a_3 \le \ldots \le a_{49}$.

Then, $y_2 = a_{25}$.

If $a_{50} \ge a_{25}$, then $y_1 \ge y_2$.

If $a_{50} \le a_{25}$, then $y_1 \le y_2$.

$\therefore$ II may NOT be true.

Since $a_{50} = x_1$, the group of numbers $\{a_1, a_2, a_3, \ldots, a_{50}\}$ is less dispersed than the group of numbers $\{a_1, a_2, a_3, \ldots, a_{49}\}$.

$\therefore$ $z_1 \le z_2$

$\therefore$ III is true.

Alternatively

$$z_1 = \frac{(a_1-x_1)^2+(a_2-x_1)^2+\cdots+(a_{49}-x_1)^2+(a_{50}-x_1)^2}{50}$$

$$= \frac{(a_1-x_1)^2+(a_2-x_1)^2+\cdots+(a_{49}-x_1)^2}{50} \quad [\because\ x_1 = a_{50}]$$

$$z_2 = \frac{(a_1-x_2)^2+(a_2-x_2)^2+\cdots+(a_{49}-x_2)^2}{49}$$

$$= \frac{(a_1-x_1)^2+(a_2-x_1)^2+\cdots+(a_{49}-x_1)^2}{49} \quad [\because\ x_1 = x_2]$$

$$\ge z_1$$

$\therefore$ III is true.