Suggested Solution for 2017 HKDSE Mathematics(core) Multiple Choice Questions

1. A

$3m^2 - 5mn + 2n^2 + m - n$

$= (3m - 2n)(m - n) + (m - n)$

$= (m - n)(3m - 2n + 1)$

2. D

$\left(\frac{1}{9^{555}}\right)3^{444}$

$=\left(\frac{1}{(3^2)^{555}}\right)3^{444}$

$=\left(\frac{1}{3^{1110}}\right)3^{444}$

$=\frac{1}{3^{666}}$

3. A

$\frac{a+4b}{2a} = 2 + \frac{b}{a}$

$\frac{a+4b}{2a} = \frac{2a+b}{a}$

$a + 4b = 2(2a + b)$

$a + 4b = 4a + 2b$

$3a = 2b$

$a = \frac{2b}{3}$

4. D

$\frac{1}{\pi^4}$

≈ 0.010265982

$= 0.010266$(correct to 6 decimal places)

5. D

$6 - x < 2x - 3$ or $7 - 3x > 1$

$6 + 3 < 2x + x$ or $7 - 1 > 3x$

$3x > 9$ or $3x < 6$

$x > 3$ or $x < 2$

i.e. $x < 2$ or $x > 3$

6. A

f(2) – f(–2)

$= 2(2)^2 - 5(2) + k - [2(-2)^2 - 5(-2) + k]$

$= -20$

7. B

p(7) = 0

$2(7)^2 - 11(7) + c = 0$

$c = -21$

The remainder

$= p\left(-\frac{1}{2}\right)$

$= 2(-\frac{1}{2})^2 - 11(-\frac{1}{2}) - 21$

$= -15$

8. A

$4x^2 + m(x + 1) + 28 \equiv mx(x + 3) + n(x - 4)$

$4x^2 + mx + m + 28 \equiv mx^2 + (3m + n)x - 4n$

By comparing coefficients of x^2,

$m = 4$

By comparing constant terms,

$-4n = m + 28$

$\quad = 4 + 28$

$n = -8$

<u>Alternatively</u>

Put $x = 4$ on both sides,

$4(4)^2 + m(4 + 1) + 28 = m(4)(4 + 3) + n(4 - 4)$

$m = 4$

Put $x = 0$ on both sides,

$4(0)^2 + 4(0 + 1) + 28 = 4(0)(0 + 3) + n(0 - 4)$

$32 = -4n$

$n = -8$

9. C

Vertex = $(-\frac{5}{p}, q)$

From the graph,

$-\frac{5}{p} < 0$

$p > 0$ and $q < 0$

10. C

Interest required

$= \$2\ 000(1 + \frac{5\%}{2})^{4\times 2} - \$2\ 000$

= \$437(correct to the nearest dollar)

11. B

Let A be the actual area of the zoo.

$\frac{4}{A} = \left(\frac{1}{20000}\right)^2$

$A = 1.6 \times 10^9 \text{ cm}^2$

$= 1.6 \times 10^5 \text{ m}^2$

12. C

Let $y = k + k_1x^2$, where k and k_1 are constants.

When $x = 1$, $y = 7$,

$7 = k + k_1(1)^2$ i.e. $k + k_1 = 7 \ldots (1)$

When $x = 2$, $y = 13$,

$13 = k + k_1(2)^2$ i.e. $k + 4k_1 = 13 \ldots (2)$

Solving (1) and (2),

$k = 5$ and $k_1 = 2$

$\therefore \quad y = 5 + 2x^2$

When $x = 3$, $y = 5 + 2(3)^2$

$= 23$

13. B

Let T(n) be the number of dots in the nth pattern. Then,

T(1) = 1

T(2) = 1 + 2(1) + 2 = 5

T(3) = 5 + 2(2) + 2 = 11

T(4) = 11 + 2(3) + 2 = 19

T(5) = 19 + 2(4) + 2 = 29

T(6) = 29 + 2(5) + 2 = 41

T(7) = 41 + 2(6) + 2 = 55

14. B

Let A cm^2 be the area of $\triangle BCD$. Then, the area of $\triangle ABD$ is $(A + 24)$ cm^2.

$A + A + 24 = \frac{1}{2} \times 14 \times 12 = 84$

$A = 30$

Then, $\frac{1}{2} \times 12 \times DC = 30$

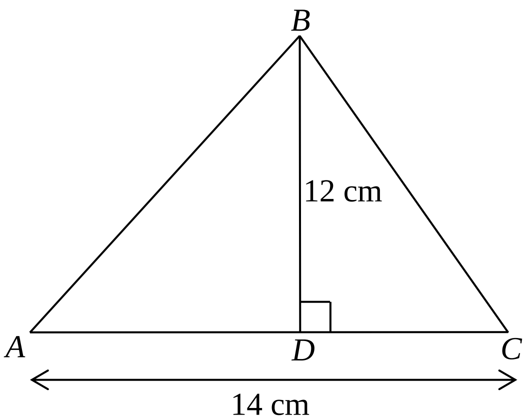
$DC = 5$ cm

$BC = \sqrt{12^2 + 5^2} = 13$ cm

$AD = 14 - 5 = 9$ cm

$AB = \sqrt{12^2 + 9^2} = 15$ cm

Perimeter of $\triangle ABC = 13 + 14 + 15 = 42$ cm



15. C

Let r cm be the base radius of the right circular cylinder and h cm be the height of the right circular cone.

$\frac{1}{3}\pi(2r)^2h = 36\pi$

$r^2h = 27$

$\therefore$ Volume of the circular cylinder

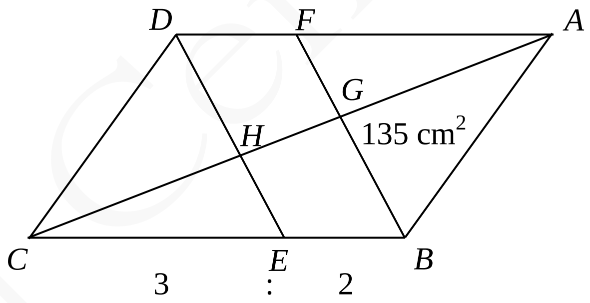
$=\pi r^2(3h)$

$= 3\pi r^2h$

$= 3\pi(27)$

$= 81\pi \text{ cm}^3$

16. D



By symmetry,

Area of $DFGH$ = area of $BEHG$

$\because$ $BE : EC = 2 : 3$

$\therefore$ $GH : HC = 2 : 3$ (intercept theorem)

and $DF : FA = BE : EC = 2 : 3$ (by symmetry)

$\therefore$ $HG : GA = DF : FA = 2 : 3$ (intercept theorem)

Then, $AG : GC = 3 : (2 + 3) = 3 : 5$

Area of $\triangle CBG$: area of $\triangle ABG = GC : AG = 5 : 3$ ($\because$ $\triangle CBG$ and $\triangle ABG$ have the same height.)

Area of $\triangle CBG : 135 = 5 : 3$

Area of $\triangle CBG = 225 \text{ cm}^2$

Note that $\triangle CEH \sim \triangle CBG$.

$\therefore$ $\frac{Area\ of\ \Delta CEH}{Area\ of\ \Delta CBG} = \left(\frac{CE}{CB}\right)^2$

i.e. $\frac{Area\ of\ \Delta CEH}{225} = \left(\frac{3}{3+2}\right)^2$

$\therefore$ Area of $\triangle CEH = 81 \text{ cm}^2$

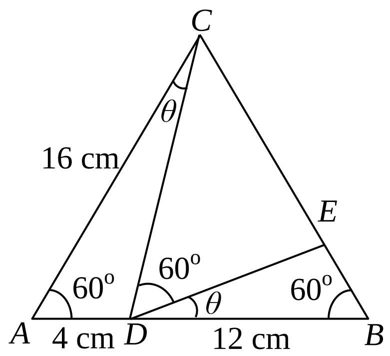
$\therefore$ Area of $BEHG$ = Area of $\triangle CBG$ – Area of $\triangle CEH$

$= 225 - 81$

$= 144 \text{ cm}^2$

i.e. Area of $DFGH = 144 \text{ cm}^2$

17. D



Note that $\triangle CAD \sim \triangle DBE$ and $DB = 12$ cm. Then,

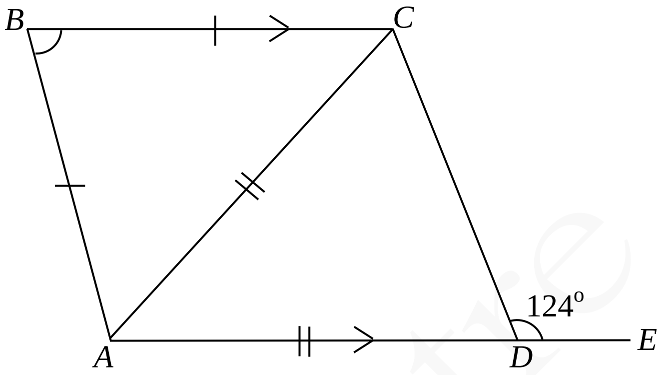
$\frac{BE}{AD} = \frac{DB}{CA}$ (corr. sides, ~$\triangle$s)

$\frac{BE}{4} = \frac{12}{16}$

$BE = 3$ cm

$CE = CB - BE = 16 - 3 = 13$ cm

18. A



$\angle CDA = 180^\circ - 124^\circ$ (adj. $\angle$s on st. line)

$= 56^\circ$

$\angle DCA = \angle CDA$ (base $\angle$s, isos. $\triangle$)

$= 56^\circ$

$\angle CAD + \angle CDA + \angle DCA = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle CAD + 56^\circ + 56^\circ = 180^\circ$

$\angle CAD = 68^\circ$

$\angle BCA = \angle CAD$ (alt. $\angle$s, AE // BC)

$= 68^\circ$

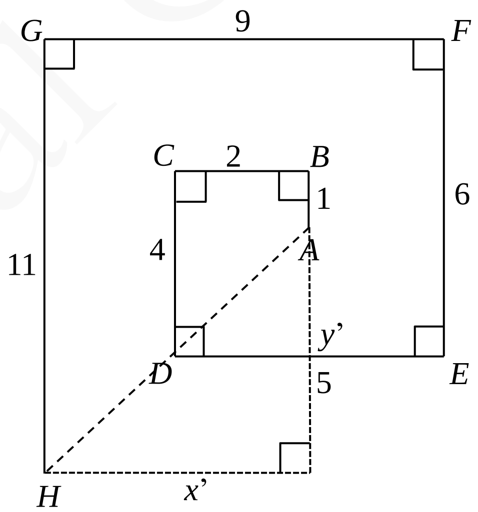
$\angle BAC = \angle BCA$ (base $\angle$s, isos. $\triangle$)

$= 68^\circ$

$\angle ABC + 68^\circ + 68^\circ = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle ABC = 44^\circ$

19. D



Vertical distance between A and H, y'

$= (11 - 6) + (4 - 1)$

$= 8$

Horizontal distance between A and H, x'

$= (9 - 5) + 2$

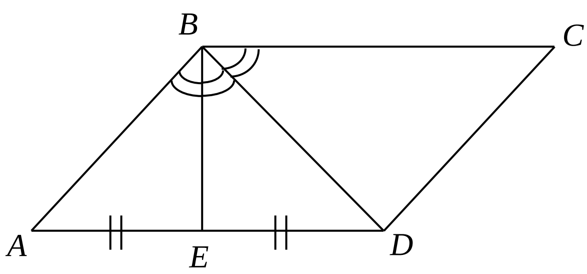
$= 6$

By Pythagoras' theorem,

$AH = \sqrt{8^2 + 6^2}$

$= 10$

20. D



Without loss of generality, $ABCD$ is drawn as shown.

$\angle BDE = \angle CBD$ (alt. $\angle$s, BC // AD)

$EB = ED$ (sides opp. equal $\angle$s)

$\angle BAE = \angle ABE$ (base $\angle$s, isos. $\triangle$)

$= \angle BDE$

Then, $AB = BD$ (sides opp. equal $\angle$s)

$\therefore$ I is true.

$\triangle ABE \cong \triangle DBE$ (SSS)

$\therefore$ III is true.

$\angle BEA = \angle BED = 90^\circ$ (adj. $\angle$s on st. line)

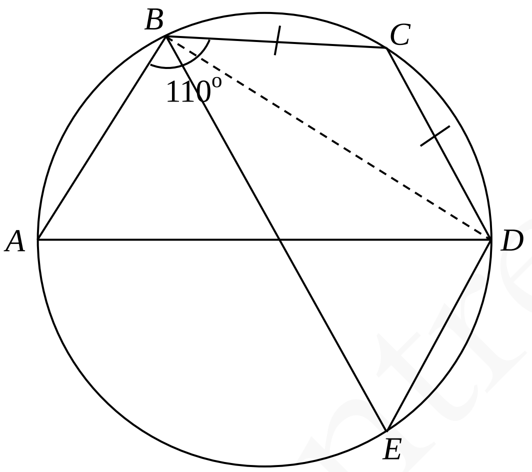
$\angle ABE + \angle BAE + \angle BEA = 180^\circ$ ($\angle$ sum of $\triangle$)

$\therefore \angle ABE = \angle BAE = 45^\circ$

$\angle ABC = \angle ABE + \angle DBE + \angle CBD = 45^\circ + 45^\circ + 45^\circ = 135^\circ$

$\therefore$ II is true.

21. C



Join BD.

$\angle ABD = 90^o$ ($\angle$ in semi-circle)

$\angle CBD = \angle ABC - \angle ABD$

$= 110^o - 90^o$

$= 20^o$

$\angle CDB = \angle CBD$ (base $\angle$s, isos. $\triangle$)

$= 20^o$

(*) $\angle BCD + \angle CBD + \angle CDB = 180^o$ ($\angle$ sum of $\triangle$)

$\angle BCD + 20^o + 20^o = 180^o$

$\angle BCD = 140^o$

$\angle BED + \angle BCD = 180^o$ (opp. $\angle$s, cyclic quad.)

$\angle BED + 140^o = 180^o$

$\angle BED = 40^o$

Alternatively

$\overset{\frown}{BC} = \overset{\frown}{CD}$ (equal chords, equal arcs)

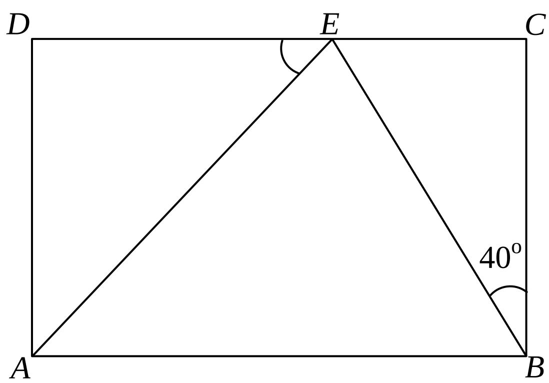
$\overset{\frown}{BC} : \overset{\frown}{BD} = 1 : 2$

$\angle BED : \angle BDC = 2 : 1$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$\angle BED : 20^o = 2 : 1$

$\angle BED = 40^o$

22. D



$\tan 40^o = \frac{EC}{2}$

$EC = 2\tan 40^o$ cm

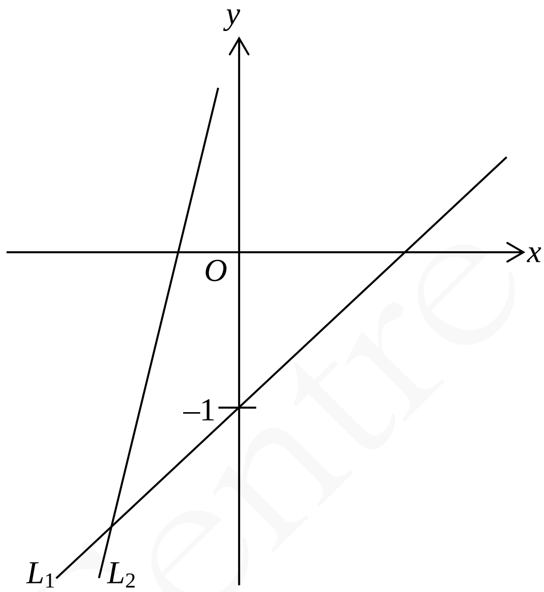
Then, $DE = (3 - 2\tan 40^o)$ cm

$\therefore \quad \tan\angle AED = \frac{AD}{DE}$

$= \frac{2}{3-2\tan 40^o}$

$\therefore \quad \angle AED \approx 57^o$ (correct to the nearest degree)

23. A



x-intercept of $L_1 = n$ and x-intercept of $L_2 = q$.

From the graphs, $n > 0 > q$ … (1)

$\therefore$ II is true.

Rewrite $L_1 : y = -\frac{1}{m}x + \frac{n}{m}$ and $L_2 : y = -\frac{1}{p}x + \frac{q}{p}$.

y-intercept of $L_1 = \frac{n}{m}$ and y-intercept of $L_2 = \frac{q}{p}$

From the graphs, $\frac{n}{m} = -1$ i.e. $n + m = 0$

$\because$ $n > 0$ [From (1)]

$\therefore$ $m < 0$ … (2)

From the graphs, $\frac{q}{p} > 0$

$\because$ $q < 0$ [From (1)]

$\therefore$ $p < 0$ … (3)

$\therefore$ $p + q < 0 = n + m$

$\therefore$ III is NOT true.

Slope of $L_1 = -\frac{1}{m}$ and slope of $L_2 = -\frac{1}{p}$.

From the graphs, $-\frac{1}{p} > -\frac{1}{m}$

$\frac{1}{p} < \frac{1}{m}$

$\because$ $m < 0$ and $p < 0$ [From (2) and (3)]

$\therefore$ $m < p$

$\therefore$ I is true.

24. A

Rewrite $y = \frac{9}{5}x + 9$ i.e. slope $= \frac{9}{5}$

Slope of $L = -1 \div \frac{9}{5} = -\frac{5}{9}$

Let the equation of L be $y = -\frac{5}{9}x + c$ where c is a constant.

Substitute $(-3, 0)$ into $y = -\frac{5}{9}x + c$,

$0 = -\frac{5}{9}(-3) + c$

$c = -\frac{5}{3}$ $\therefore$ The required equation is $y = -\frac{5}{9}x - \frac{5}{3}$ i.e. $5x + 9y + 15 = 0$.

25. C

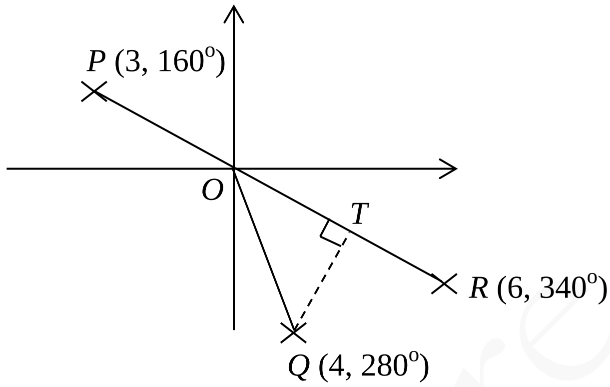
Let T be the foot of the perpendicular from Q to PR.

Note that $\angle QOT = 60^\circ$.

$\frac{QT}{OQ} = \sin \angle QOT$

$\frac{QT}{4} = \sin 60^\circ$

$QT = 2\sqrt{3}$



26. A

Rewrite the equation of the circle C_2 as $x^2 + y^2 + 4x - 2y - \frac{5}{2} = 0$.

$G_1 = (-4, 2)$, $G_2 = (-2, 1)$

Slope of $OG_1 = \frac{2}{-4} = -\frac{1}{2}$ and slope of $OG_2 = \frac{1}{-2}$ = slope of OG_1

$\therefore$ G_1, G_2 and O are collinear.

$\therefore$ I is true.

Radius of C_1, $r_1 = \sqrt{(-4)^2 + (2)^2 - (-5)}$
$= 5$

Radius of C_2, $r_2 = \sqrt{(-2)^2 + (1)^2 - (-\frac{5}{2})}$
$= \sqrt{\frac{15}{2}} \neq r_1$

$\therefore$ II is NOT true.

$OG_1 = \sqrt{(-4)^2 + (2)^2}$
$= \sqrt{20}$

$OG_2 = \sqrt{(-2)^2 + (1)^2}$
$= \sqrt{5}$
$\neq OG_1$

$\therefore$ III is NOT true.

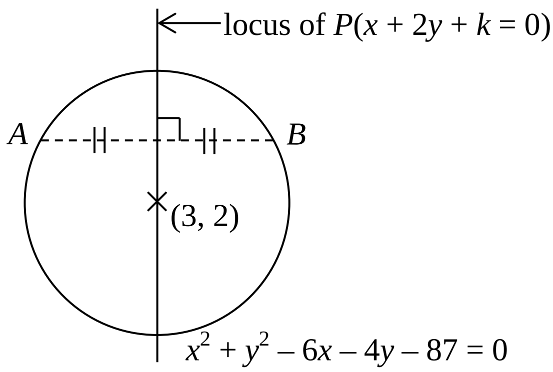
27. B

Note that the locus of P is the perpendicular bisector of AB and passes through the centre of the circle.

Centre of the circle = (3, 2)

$\therefore$ $(3) + 2(2) + k = 0$

$k = -7$



28. C

Number of favourable outcomes = 5 + 20 = 25

Number of possible outcomes = 5 + 20 + 15 + 10 + 10 = 60

$\therefore$ The required probability $= \frac{25}{60}$

$= \frac{5}{12}$

29. B

The lower quartile = the lower end of the box = 15

30. B

$\because$ Mean = 5

$\therefore$ $\frac{2+3+4+6+7+9+10+m+n}{9} = 5$

i.e. $m + n = 4$

When $m = 1$ and $n = 3$,

$a = 3$, $b = 4$ and $c = 10 - 1 = 9$.

When $m = 2$ and $n = 2$,

$a = 2$, $b = 4$ and $c = 10 - 2 = 8$.

$\therefore$ Only $b = 4$ must be true.

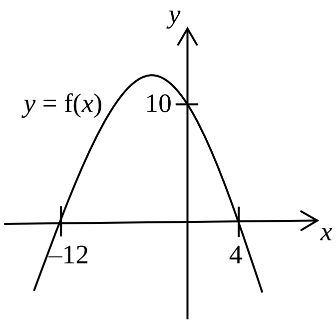
31. D

Note that $g(x) = f\left(\frac{x}{2}\right)$ represents an enlargement of 2 times along the x-axis. The answer is D.

<u>Alternatively</u>

From the graph of f(x),

$f(0) = 10$, $f(-12) = f(4) = 0$



$\because$ $g(x) = f\left(\frac{x}{2}\right)$

$\therefore$ $g(0) = f\left(\frac{0}{2}\right) = f(0) = 10$ i.e. The y-intercept of g(x) is 10.

$g(-24) = f\left(\frac{-24}{2}\right) = f(-12) = 0$ and $g(8) = f\left(\frac{8}{2}\right) = f(4) = 0$ i.e. The x-intercepts of g(x) are –24 and 8.

32. D

Note that $8^3 = 2(16^2)$ and $8^4 = 16^3$.

$8^3 + 8^{19}$

$= 8^3(1 + 8^{16})$

$= 2(16^2)[1 + (8^4)^4]$

$= 2(16^2)[1 + (16^3)^4]$

$= 2(16^2)(1 + 16^{12})$

$= 2(16^2) + 2(16^{14})$

$= 200000000000200_{16}$

16^{14}	16^{13}	16^{12}	16^{11}	16^{10}	16^9	16^8	16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0
2	0	0	0	0	0	0	0	0	0	0	0	2	0	0

33. C

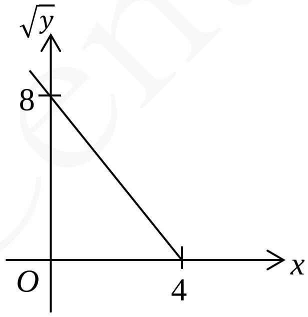
Slope of the graph = –2 and $\sqrt{y}$ – intercept of the graph = 8

Equation of the straight line is

$\sqrt{y} = -2x + 8$

$y = (-2x + 8)^2$

$\quad = 4x^2 - 32x + 64$



34. D

$$\begin{cases} \log_9 y = x - 3 \quad \text{i.e. } x = \log_9 y + 3 \ \ldots (1) \\ 2(\log_9 y)^2 = 4 - x \ \ldots (2) \end{cases}$$

Substitute (1) into (2),

$2(\log_9 y)^2 = 4 - (\log_9 y + 3)$

$2(\log_9 y)^2 = 4 - \log_9 y - 3$

$2(\log_9 y)^2 + \log_9 y - 1 = 0$

$(2\log_9 y - 1)(\log_9 y + 1) = 0$

$\log_9 y = \frac{1}{2}$ or -1

$y = 3$ or $\frac{1}{9}$

35. B

$\frac{5}{2-i} + ki$

$= \frac{5(2+i)}{(2-i)(2+i)} + ki$

$= \frac{10+5i}{2^2-i^2} + ki$

$= 2 + (k + 1)i$

$\because$ The number is a real number.

$\therefore$ $k + 1 = 0$

$k = -1$

36. C

$\pi^{45} - \pi^{30} \neq \pi^{60} - \pi^{45}$

∴ I is NOT an arithmetic sequence.

$45\pi - 30\pi = 60\pi - 45\pi = 15\pi$

∴ II is an arithmetic sequence.

$(\pi - 45) - (\pi - 30) = (\pi - 60) - (\pi - 45) = -15$

∴ III is an arithmetic sequence.

37. C

Draw the straight lines of $y = 9$, $x - y - 9 = 0$ and $x + y - 9 = 0$.

The points of intersections are (0, 9), (9, 0) and (18, 9).

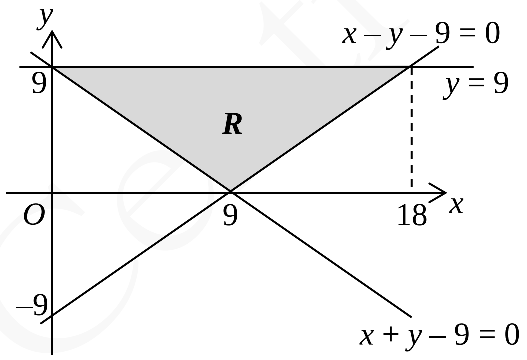
Let $P = x - 2y + 43$.

$P(0, 9) = (0) - 2(9) + 43 = 25$

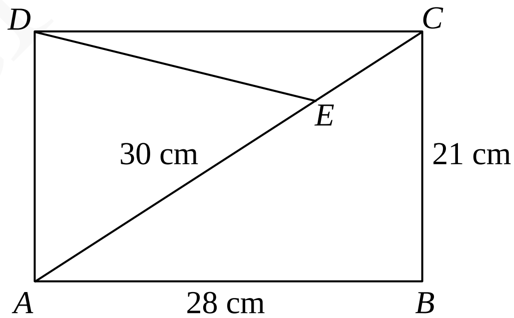
$P(9, 0) = (9) - 2(0) + 43 = 52$

$P(18, 9) = (18) - 2(9) + 43 = 43$

∴ The greatest value of $x - 2y + 43$ is 52.



38. A



By Pythagoras' theorem,

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{28^2 + 21^2}$

$= 35$ cm

$EC = AC - AE$

$= 35 - 30$

$= 5$ cm

$\cos\angle BAC = \frac{AB}{AC} = \frac{28}{35} = \frac{4}{5}$

Note that $\angle DCA = \angle BAC$ (alt. ∠s, $AB//DC$)

∴ $\cos\angle DCA = \cos\angle BAC = \frac{4}{5}$

By cosine formula,

$DE^2 = CD^2 + EC^2 - 2(CD)(EC)\cos\angle DCA$

$= 28^2 + 5^2 - 2(28)(5)\left(\frac{4}{5}\right)$

$= 585$

$DE = \sqrt{585}$

$= 3\sqrt{65}$ cm

<u>Alternatively</u>

$\cos\angle BCA = \frac{BC}{AC} = \frac{21}{35} = \frac{3}{5}$

Note that $\angle DAC = \angle BCA$ (alt. ∠s, $AD//BC$)

∴ $\cos\angle DAC = \cos\angle BCA = \frac{3}{5}$

By cosine formula,

$DE^2 = AD^2 + AE^2 - 2(AD)(AE)\cos\angle DAC$

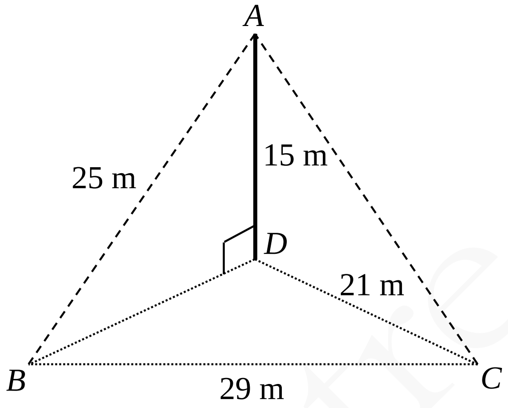
$= 21^2 + 30^2 - 2(21)(30)\left(\frac{3}{5}\right)$

$= 585$

$DE = \sqrt{585}$

$= 3\sqrt{65}$ cm

39. A



By Pythagoras' theorem,

$BD^2 + AD^2 = AB^2$

$BD^2 + 15^2 = 25^2$

$BD = 20$ m

$BD^2 + CD^2 = 20^2 + 21^2$

$= 29^2$

$= BC^2$

$\therefore$ $\triangle BCD$ is a right-angled $\triangle$ with $\angle BDC = 90^\circ$.

i.e. Plane ABD is perpendicular to plane ADC.

The projection of AB on plane ADC is AD. The required angle is $\angle BAD$. Then,

$\tan\angle BAD = \frac{20}{15}$ /$\sin\angle BAD = \frac{20}{25}$ /$\cos\angle BAD = \frac{15}{25}$

$\therefore$ $\angle BAD = 53^\circ$ (correct to the nearest degree)

40. B

$\angle OAD = 90^\circ$ (tangent $\perp$ radius)

$\angle OAB = \angle OAD - \angle BAD$

$= 90^\circ - 68^\circ$

$= 22^\circ$

Join BO.

$\angle OBA = \angle OAB$ (base $\angle$s, isos. $\triangle$)

$= 22^\circ$

$\angle CBO = \angle BCO$ (base $\angle$s, isos. $\triangle$)

$= 26^\circ$

$\angle ABC = \angle CBO + \angle OBA$

$= 26^\circ + 22^\circ$

$= 48^\circ$

<u>Alternatively</u>

Join AC.

$\angle ACB = \angle BAD$ ($\angle$ in alt. segment)

$= 68^\circ$

$\angle OCA = \angle ACB - \angle BCO$

$= 68^\circ - 26^\circ$

$= 42^\circ$

$\angle OAC = \angle OCA$ (base $\angle$s, isos. $\triangle$)

$= 42^\circ$

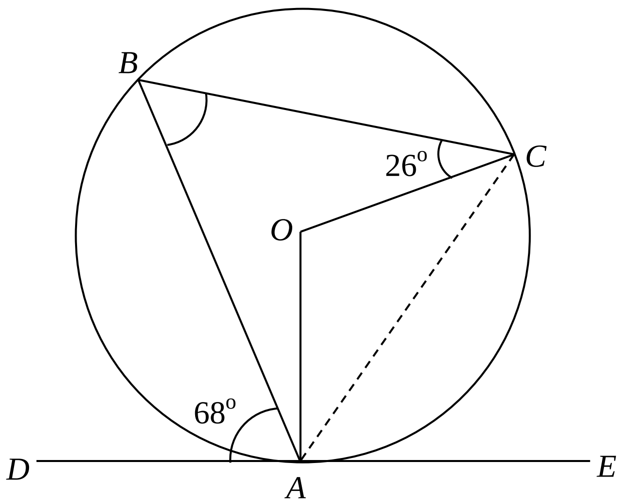
$\angle AOC + \angle OAC + \angle OCA = 180^\circ$ ($\angle$ sum of $\triangle$)

$\angle AOC + 42^\circ + 42^\circ = 180^\circ$

$\angle AOC = 96^\circ$

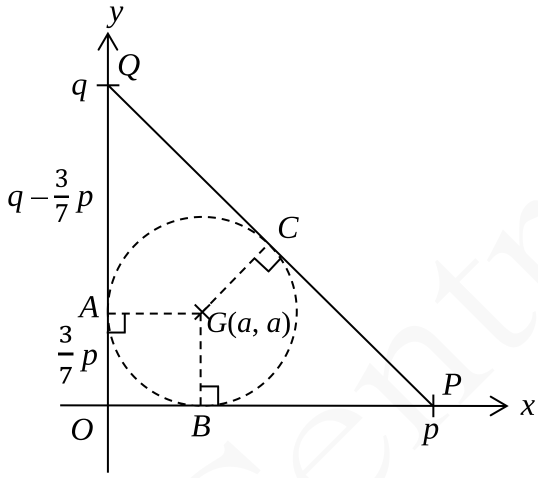
$\angle ABC = \frac{1}{2}\angle AOC$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$= 48^\circ$



41. D

Let $G(a, a)$ be the in-centre of $\triangle OPQ$ and A, B and C be the tangent points to the circle inscribed in $\triangle OPQ$ on the y-axis, the x-axis and PQ respectively. Then,



$3a + 4a = 3p$

$a = \frac{3}{7}p$

$OA = OB = a = \frac{3}{7}p$ (tangent properties)

$QC = QA = q - \frac{3}{7}p$ (tangent properties)

$CP = BP = p - \frac{3}{7}p$ (tangent properties)

$= \frac{4}{7}p$

$GC = a = \frac{3}{7}p$

Note that $\triangle QAG \cong \triangle QCG$ and $\triangle PBG \cong \triangle PCG$.

Area of $\triangle OPQ$ = area of $AOBG$ + 2 × area of $\triangle QAG$ + 2 × area of $\triangle PBG$

$\frac{1}{2}pq = (\frac{3}{7}p)(\frac{3}{7}p) + 2 \times \frac{1}{2} \times (q - \frac{3}{7}p)(\frac{3}{7}p) + 2 \times \frac{1}{2} \times (\frac{4}{7}p)(\frac{3}{7}p)$

$\frac{p}{q} = \frac{7}{24}$ i.e. $p : q = 7 : 24$

<u>Alternatively</u>

$QP = QC + CP$

$QP^2 = (QC + CP)^2$

$p^2 + q^2 = (q - \frac{3}{7}p + \frac{4}{7}p)^2$

$p^2 + q^2 = q^2 + \frac{2}{7}pq + \frac{1}{49}p^2$

$\frac{p}{q} = \frac{7}{24}$ i.e. $p : q = 7 : 24$

42. B

Number of teams formed

$= C_5^{13} \times C_4^6$

$= 19\ 305$

43. C

The required probability = 1 – P("she hits the target 4 times")

$= 1 - (0.7)^4$

$= 0.7599$

44. B

Let σ be the standard deviation of the test scores. Then,

$$\frac{33-45}{\sigma} = -2$$

$\sigma = 6$

∴ The standard deviation of the scores is 6 marks.

45. A

As all data are increased to 8 times the original ones, the new mode is increased to 8 times the original one as well.

∴ I is true.

As all data are increased to 8 times the original ones, the new inter-quartile range is increased to 8 times the original one as well.

∴ II is true.

As all data are increased to 8 times the original ones, the new variance is increased to 64 times the original one.

∴ III is NOT true.