Suggested Solution for 2019 HKDSE Mathematics(core) Multiple Choice Questions

1. C

$(a-b)(a^2+ab-b^2)$
$= a(a^2+ab-b^2) - b(a^2+ab-b^2)$
$= a^3 + a^2b - ab^2 - a^2b - ab^2 + b^3$
$= a^3 - 2ab^2 + b^3$

2. D

$\frac{(6x^7)^2}{4x^5}$
$= \frac{6^2x^{7\times2}}{4x^5}$
$= \frac{36x^{14}}{4x^5}$
$= 9x^9$

3. B

$\begin{cases} 6x - 7y = 40 \ \ldots\ (1) \\ 2x + 11y = 40 \ \ldots\ (2) \end{cases}$
$(2) \times 3 - (1)$,
$40y = 80$
$y = 2$

4. C

$(x-8)(x+\alpha) - 6 \equiv (x-9)^2 + \beta$
Substitute $x = 8$ into both sides,
$(8-8)(8+\alpha) - 6 = (8-9)^2 + \beta$
$-6 = 1 + \beta$
$\beta = -7$

Alternatively
$x^2 + (\alpha - 8)x - 8\alpha - 6 \equiv x^2 - 18x + 81 + \beta$
By comparing the coefficients of x,
$\alpha - 8 = -18$
$\alpha = -10$
By comparing the constant terms,
$81 + \beta = -8\alpha - 6 = -8(-10) - 6$
$\beta = -7$

5. A

$h = 3 - \frac{5}{k+4}$
$h(k+4) = 3(k+4) - 5$
$hk + 4h = 3k + 12 - 5$
$4h - 7 = -3k - hk$
$k(3-h) = 4h - 7$
$k = \frac{4h-7}{3-h}$

6. D

$x = 0.07$(correct to 2 decimal places)

$x = 0.066$(correct to 2 significant figures)

$x = 0.066$(correct to 3 decimal places)

$x = 0.0656$(correct to 3 significant figures)

7. B

$-2(x-5)+5<21$ or $\frac{3x-5}{7}>1$

$-2x+10+5<21$ or $3x-5>7$

$-2x<21-15$ or $3x>7+5$

$-2x<6$ or $3x>12$

$x>-3$ or $x>4$

$\therefore$ $x>-3$

$\therefore$ The least integer is –2.

8. C

$f(c)+f(-c)$

$=(c)^3+c(c)^2+c+(-c)^3+c(-c)^2+c$

$=c^3+c^3+c-c^3+c^3+c$

$=2c^3+2c$

9. D

By Factor theorem,

$2\left(-\frac{k}{2}\right)^4+k\left(-\frac{k}{2}\right)^3-4\left(-\frac{k}{2}\right)-16=0$

$\frac{k^4}{8}-\frac{k^4}{8}+2k-16=0$

$k=8$

10. A

$y=(3-x)(x+2)+6$

$=-x^2+x+6+6$

$=-x^2+x+12$

$\because$ $a=-1<0$

$\therefore$ The graph opens downwards.

$\therefore$ I is true.

Put $x=1$,

$-(1)^2+(1)+12=12\neq 10$

$\therefore$ II is NOT true.

When $y=0$,

$-x^2+x+12=0$

$x^2-x-12=0$

$(x+3)(x-4)=0$

$x=-3$ or 4

i.e. The x-intercepts of the graph are –3 and 4.

$\therefore$ III is NOT true.

11. C

The required amount = $\$65\ 000(1 + \frac{7\%}{4})^{8 \times 4}$

= \$113 244(correct to the nearest dollar)

12. B

$\frac{140x+315y}{x+y} = 210$

$140x + 315y = 210(x + y)$

$140x + 315y = 210x + 210y$

$210x - 140x = 315y - 210y$

$70x = 105y$

$\frac{x}{y} = \frac{105}{70} = \frac{3}{2}$

i.e. $x : y = 3 : 2$

13. A

$z = \frac{kx^2}{\sqrt{y}}$ where k is a constant.

Change in z, $z' = \frac{k[(1-40\%)x]^2}{\sqrt{(1+44\%)y}}$

$= \frac{0.3kx^2}{\sqrt{y}}$

$= 0.3z$

Percentage change in z

$= \frac{0.3z - z}{z} \times 100\%$

$= -70\%$

14. C

T(1) = 6

T(2) = 10

T(3) = 14

:

:

T(n) = 4n + 2

∴ T(9) = 4(9) + 2

= 38

15. D

Let h cm be the height of the lateral surface.

By Pythagoras' theorem,

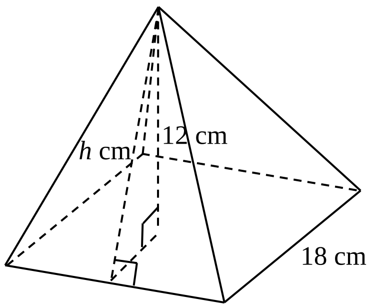
$h = \sqrt{12^2 + 9^2}$

$= 15$ cm

The total surface area of the pyramid

$= 18 \times 18 + \frac{1}{2} \times 18 \times 15 \times 4$

$= 864$ cm^2



16. D

Note that $\triangle BEX \sim \triangle BFY \sim \triangle DAX$.

$\therefore$ $EX : FY = BE : BF = 2 : 9$ and

$EX : AX = BE : DA = 2 : 12 = 1 : 6$

$\because$ $\triangle BEX$ and $\triangle ABX$ have the same height.

$\therefore$ Area of $\triangle BEX$: area of $\triangle ABX = EX : AX$

Area of $\triangle BEX : 24 = 1 : 6$

Area of $\triangle BEX = 4$ cm^2

$\because$ $\triangle BEX \sim \triangle BFY$

$\therefore$ $\frac{Area\ of\ \Delta BEX}{Area\ of\ \Delta BFY} = \left(\frac{EX}{FY}\right)^2$

i.e. $\frac{4}{Area\ of\ \Delta BFY} = \left(\frac{2}{9}\right)^2$

$\therefore$ Area of $\triangle BFY = 81$ cm^2

$\because$ $\triangle BEX \sim \triangle DAX$

$\therefore$ $\frac{Area\ of\ \Delta DAX}{Area\ of\ \Delta BEX} = \left(\frac{DA}{BE}\right)^2$

i.e. $\frac{Area\ of\ \Delta DAX}{4} = \left(\frac{6}{1}\right)^2$

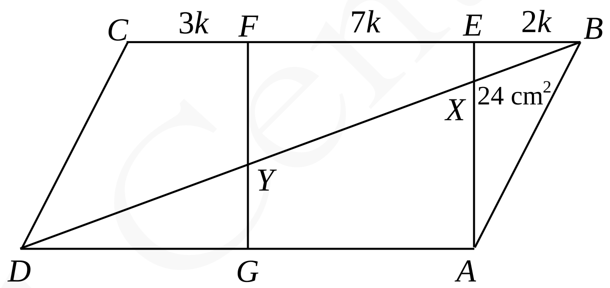
Area of $\triangle DAX = 144$ cm^2

Area of $CDYF$ = Area of $\triangle BDC$ – Area of $\triangle BFY$

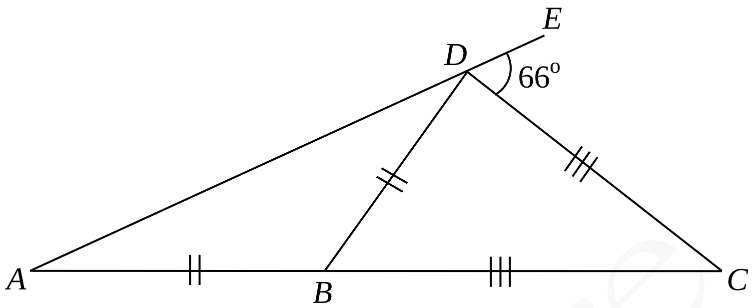
= Area of $\triangle DAX$ + Area of $\triangle ABX$ – Area of $\triangle BFY$

$= 144 + 24 - 81$

$= 87$ cm^2



17. A



$\angle BAD = \angle BDA$ (base $\angle$s, isos. $\triangle$)

Let $\angle BAD = \angle BDA = x$.

$\angle CBD = \angle BAD + \angle BDA$ (ext. $\angle$ of $\triangle$)

$= x + x$

$= 2x$

$\angle CDB = \angle CBD$ (base $\angle$s, isos. $\triangle$)

$= 2x$

$\angle BDA + \angle CDB + \angle CDE = 180^o$ (adj. $\angle$s on st. line)

$x + 2x + 66^o = 180^o$

$x = 38^o$

$\angle BAD + \angle ACD = \angle CDE$ (ext. $\angle$ of $\triangle$)

$38^o + \angle ACD = 66^o$

$\angle ACD = 28^o$

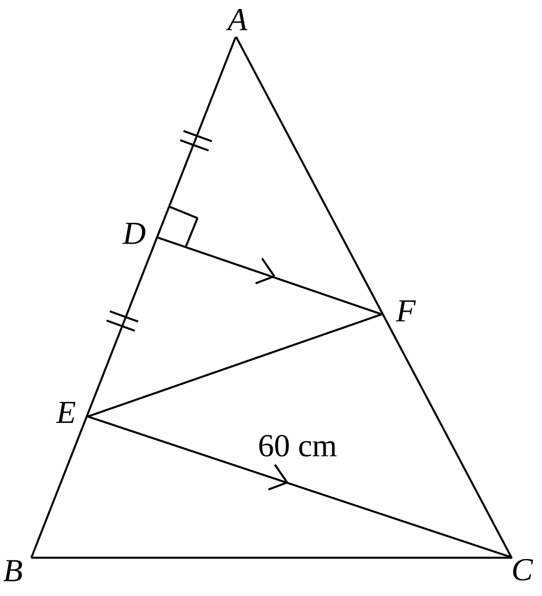
<u>Alternatively</u>

$\angle CDB + \angle CBD + \angle ACD = 180^o$ ($\angle$ sum of $\triangle$)

$2(38^o) + 2(38^o) + \angle ACD = 180^o$

$\angle ACD = 28^o$

18. D



Let $EB = x$ cm. Then, $AD = DE = 2x$ cm.

$AC = AB = 5x$ cm

$\angle AEC = \angle ADF$ (corr. $\angle$s, $DF // EC$)

$= 90^o$

By Pythagoras' theorem,

$AE^2 + EC^2 = AC^2$

$(4x)^2 + 60^2 = (5x)^2$

$x = 20$

By mid-point theorem,

$FD = \frac{1}{2}CE = 30$ cm

By Pythagoras' theorem,

$EF^2 = FD^2 + DE^2$

$= 30^2 + 40^2$

$EF = 50$ cm

19. A

By Pythagoras' theorem,

$AB^2 + BD^2 = AD^2$

$18^2 + BD^2 = 30^2$

$BD = 24$ cm

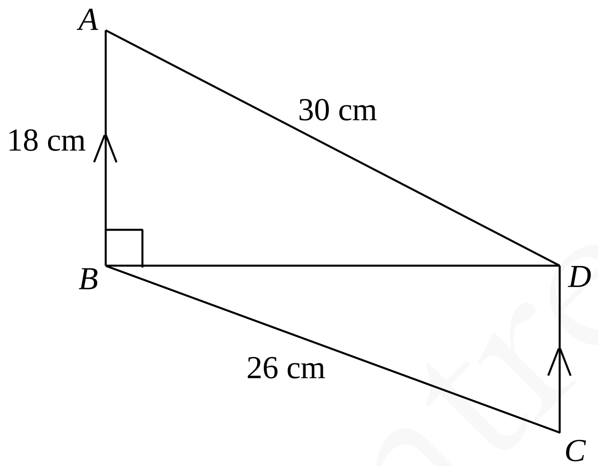
$\angle CDB = \angle ABD = 90^o$ (alt. ∠s, $AB//DC$)

By Pythagoras' theorem,

$DC^2 + BD^2 = BC^2$

$DC^2 + 24^2 = 26^2$

$DC = 10$ cm



∴ Area of $ABCD = \frac{1}{2} \times 18 \times 24 + \frac{1}{2} \times 10 \times 24$ / $\frac{1}{2} \times (18 + 10) \times 24$

$= 336 \text{ cm}^2$

20. C

$\angle EBF = \angle EFB$ (base ∠s, isos. △)

$\angle EBF + \angle EFB + \angle BEF = 180^o$

$2\angle EBF + 56^o = 180^o$

$\angle EBF = 62^o$

$\angle DBC = \angle ABD$ (properties of rhombus)

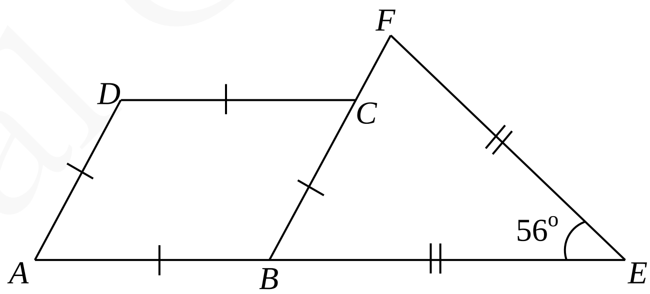
$\angle DBC + \angle ABD + \angle EBF = 180^o$ (adj. ∠s on st. line)

$2\angle ABD + 62^o = 180^o$

$\angle ABD = 59^o$

$\angle BDC = \angle ABD$ (alt. ∠s, $AB//DC$)

$= 59^o$



21. B

$\angle AOC = \angle BOD$ (equal chords, equal ∠s)

$\angle AOC = \angle AOD + \angle COD$

$= \angle AOD + 48^o$

Similarly,

$\angle BOD = \angle BOC + \angle COD$

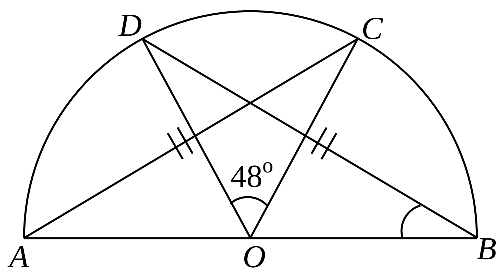
$= \angle BOC + 48^o$

∴ $\angle AOD + 48^o = \angle BOC + 48^o$ i.e. $\angle AOD = \angle BOC$

$\angle AOD + \angle COD + \angle BOC = 180^o$ (adj. ∠s on st. line)

$2\angle AOD + 48^o = 180^o$

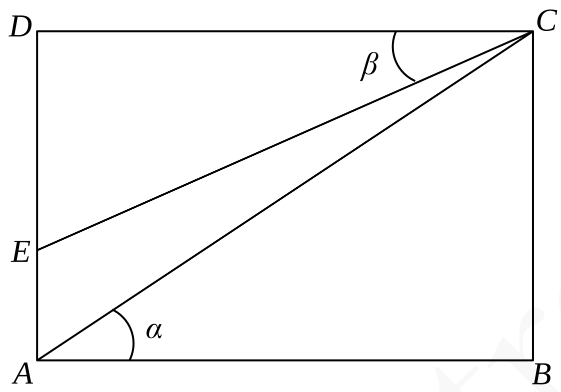
$\angle AOD = 66^o$



$\angle ABD = \frac{1}{2}\angle AOD$ (∠ at centre twice ∠ at ⊙ce)

$= 33^o$

22. B



$\frac{AB}{AC} = \cos\alpha$ i.e. $AB = AC\cos\alpha$

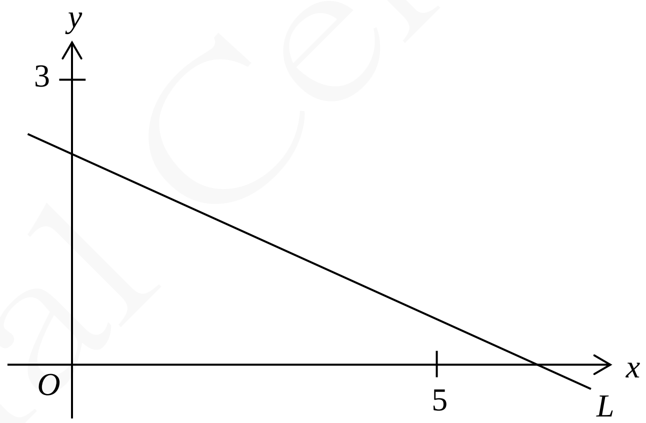
$\frac{CD}{CE} = \cos\beta$ i.e. $CD = CE\cos\beta$

$\because$ $AB = CD$

$\therefore$ $AC\cos\alpha = CE\cos\beta$

i.e. $\frac{CE}{AC} = \frac{\cos\alpha}{\cos\beta}$

23. A



Put $y = 0$,

x–intercept = $-\frac{15}{a}$

From the graph, $-\frac{15}{a} > 5$

$\therefore$ $-3 < a < 0$

$\therefore$ II is true.

Put $x = 0$,

y–intercept = $-\frac{15}{b}$

From the graph, $0 < -\frac{15}{b} < 3$

$\therefore$ $b < -5$

$\therefore$ III is NOT true.

$\because$ $-3 < a < 0$ and $b < -5$

$\therefore$ $a > b$

$\therefore$ I is true.

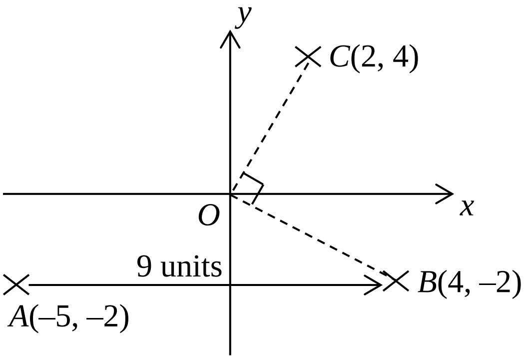
24. A

Rewrite the equations of the straight lines as $y = -\frac{3}{2}x - \frac{k}{2}$ and $y = -\frac{k}{12}x + \frac{1}{2}$.

$\therefore$ $-\frac{3}{2} \times -\frac{k}{12} = -1$

$k = -8$

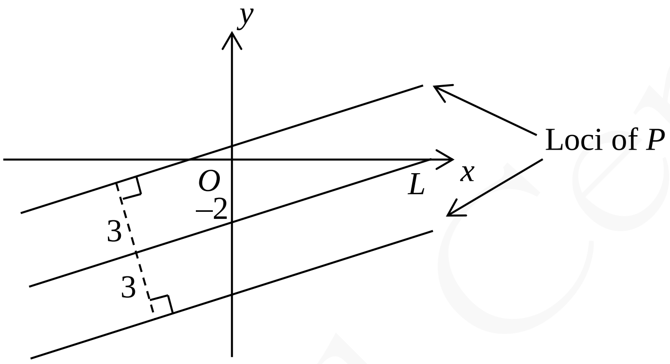
25. D



26. D

Rewrite the equation of L as

$y = \frac{5}{7}x - 2$.



27. B

Rewrite the equation of the circle C as $x^2 + y^2 + 2x - 6y + \frac{15}{2} = 0$.

Centre of $C = (-1, 3)$

∴ III is NOT true.

Radius of C, $r = \sqrt{(-1)^2 + (3)^2 - (\frac{15}{2})}$

$= \frac{\sqrt{10}}{2}$

The area of $C = \pi r^2$

$= \pi\left(\frac{\sqrt{10}}{2}\right)^2$

$= \frac{5\pi}{2}$

∴ I is NOT true.

Distance between the centre[(–1, 3)] and (–3, 3)

$= 2 > \frac{\sqrt{10}}{2}$

i.e. (–3, 3) lies outside C.

∴ II is true.

28. C

Favourable outcomes include (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8) and (8, 9). There are 8 favourable outcomes.

Number of possible outcomes = C_2^9

∴ The required probability = $\frac{8}{C_2^9}$

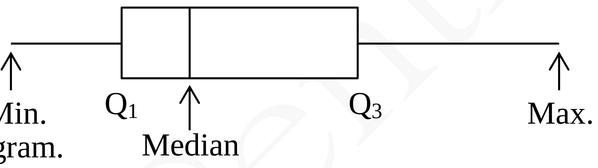
$= \frac{2}{9}$

29. B

Range = Maximum value – minimum value

Inter-quartile range = $Q_3 - Q_1$

∴ I and III can be obtained from a box-and-whisker diagram.



30. C

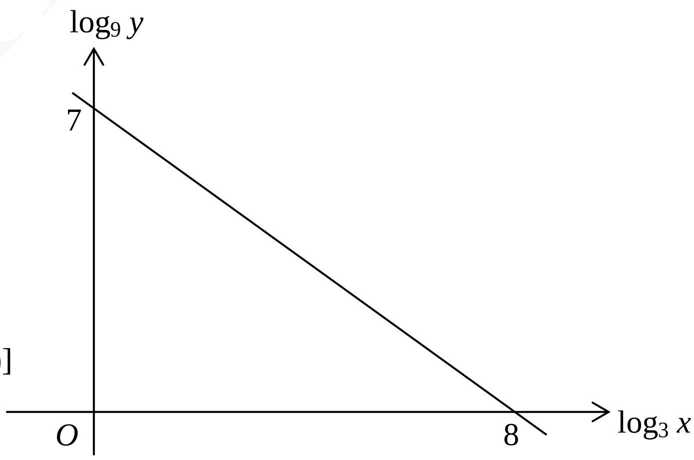
There are 116 students.

Mode = median = 7

The lower quartile = 6

The upper quartile = 8

31. B



Slope = $\frac{7-0}{0-8} = -\frac{7}{8}$

$\log_9 y = -\frac{7}{8}\log_3 x + 7$ [c.f. $y = mx + c$]

$= \log_3 x^{-\frac{7}{8}} + \log_3 3^7$ [use $x\log y = \log y^x$]

$= \log_3 (3^7 \bullet x^{-\frac{7}{8}})$ [use $\log x + \log y = \log (xy)$]

$\frac{\log y}{\log 9} = \frac{\log(3^7 \cdot x^{-\frac{7}{8}})}{\log 3}$ [use change of base formula]

$\frac{\log y}{2\log 3} = \frac{\log(3^7 \cdot x^{-\frac{7}{8}})}{\log 3}$

$\log y^{\frac{1}{2}} = \log (3^7 \bullet x^{-\frac{7}{8}})$

$y^{\frac{1}{2}} = 3^7 \bullet x^{-\frac{7}{8}}$

$x^{\frac{7}{8}}y^{\frac{1}{2}} = 3^7$

$\left(x^{\frac{7}{8}}y^{\frac{1}{2}}\right)^8 = (3^7)^8$ i.e. $x^7y^4 = 3^{56}$

32. D

$\frac{3}{3\log x-2}$ + 7 = $\frac{2}{2\log x+1}$

3(2log x + 1) + 7(3log x – 2)(2log x + 1) = 2(3log x – 2)

6log x + 3 + 42(log x)^2 – 7log x – 14 = 6log x – 4

6(log x)^2 – log x – 1 = 0

(2log x – 1)(3log x + 1) = 0 [By letting u = log x, this is like (2u – 1)(3u + 1) = 0]

log x = $\frac{1}{2}$ or $-\frac{1}{3}$

∴ log $\frac{1}{x}$ = log 1 – log x [use log $\frac{x}{y}$ = log x – log y]

= – log x

= $\frac{-1}{2}$ or $\frac{1}{3}$

33. A

2^{14}	2^{13}	2^{12}	2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	1	1	0	0	0	0	0	1	0	1	1	0

100110000010110_2

= $2^{14} + 2^{11} + 2^{10} + 2^4 + 2^2 + 2$

= $(2^4 + 2^1 + 1) \times 2^{10} + 16 + 4 + 2$

= $(16 + 2 + 1) \times 2^{10} + 22$

= $19 \times 2^{10} + 22$

34. D

$\frac{4+i^5}{a+i} - i^6$

$= \frac{4+i}{a+i} - i^2$ [∵ $i^4 = 1$]

$= \frac{4+i}{a+i} \times \frac{a-i}{a-i} - (-1)$

$= \frac{4a+ai-4i-i^2}{a^2-i^2} + 1$

$= \frac{4a+ai-4i-(-1)}{a^2-(-1)} + 1$

$= \frac{4a+1+(a-4)i}{a^2+1} + 1$

$= \frac{4a+1+(a-4)i+a^2+1}{a^2+1}$

$= \frac{a^2+4a+2}{a^2+1} + \frac{a-4}{a^2+1}i$ i.e. The real part is $\frac{a^2+4a+2}{a^2+1}$.

35. C

Draw the straight lines of $x + 2y = 20$, $7x - 6y = 20$ and $13x + 6y = 20$.

The points of intersections are $(2, -1)$, $(-4, 12)$ and $(8, 6)$.

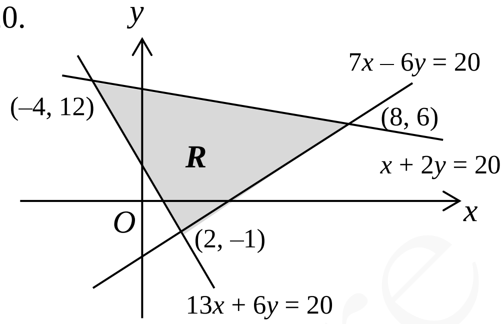
Let $P = 7x + 8y + 9$.

$P(2, -1) = 15$

$P(-4, 12) = 77$

$P(8, 6) = 113$

$\therefore$ The greatest value of $7x + 8y + 9$ is 113.



36. C

Let a be the first term of the geometric sequence and r be the common ratio.

$ar + ar^4 = 9$ i.e. $ar^6 + ar^9 = 9r^5$ … (1)

$ar^6 + ar^9 = 288$ … (2)

Combining (1) and (2),

$\therefore$ $9r^5 = 288$

We get $r = 2$ and $a = \frac{1}{2}$

$\therefore$ 20^{th} term $= ar^{19}$

$= \left(\frac{1}{2}\right)(2)^{19}$

$= 262\,144$

37. A

$$\begin{cases} 3x - y - 2 = 0 & \text{i.e. } y = 3x - 2 \text{ … (1)} \\ 5x^2 + 5y^2 + kx + 4y - 20 = 0 \text{ … (2)} \end{cases}$$

Substitute (1) into (2),

$5x^2 + 5(3x - 2)^2 + kx + 4(3x - 2) - 20 = 0$

$5x^2 + 5(9x^2 - 12x + 4) + kx + 12x - 8 - 20 = 0$

$50x^2 + (k - 48)x - 8 = 0$

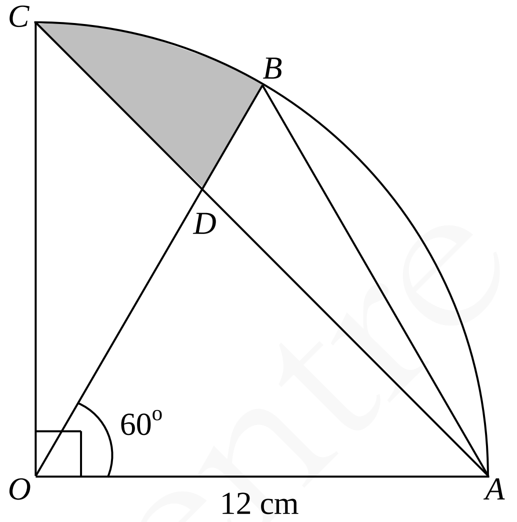
The x-coordinate of the mid-point of $PQ = \frac{sum\ of\ roots}{2}$

$= \frac{-\frac{k-48}{50}}{2}$

$= 2$

$\therefore$ $k = -152$

38. A



∵ $\triangle OAB$ is an equilateral $\triangle$.

∴ $\angle BOA = 60^o$

$\angle OCD = \angle OAD$ (base $\angle$s, isos. $\triangle$)

$\angle OCD + \angle OAD + \angle AOC = 180^o$ ($\angle$ sum of $\triangle$)

$2\angle OAD + 90^o = 180^o$

$\angle OAD = 45^o$

∴ $\angle OCD = \angle OAD = 45^o$

$\angle ODC = \angle OAD + \angle DOA$ (ext. $\angle$ of $\triangle$)

$= 45^o + 60^o = 105^o$

By sine formula,

$$\frac{OD}{\sin\angle OCD} = \frac{OC}{\sin\angle ODC}$$

$$\frac{OD}{\sin 45^o} = \frac{12}{\sin 105^o}$$

i.e. $OD = \dfrac{12\sin 45^o}{\sin 105^o}$ cm

$\angle COD = 90^o - 60^o = 30^o$

Area of $\triangle COD$

$= \dfrac{1}{2} \times OC \times OD \times \sin\angle COD$

$= \dfrac{1}{2} \times 12 \times \dfrac{12\sin 45^o}{\sin 105^o} \times \sin 30^o$

$= \dfrac{36\sin 45^o}{\sin 105^o}$ cm^2

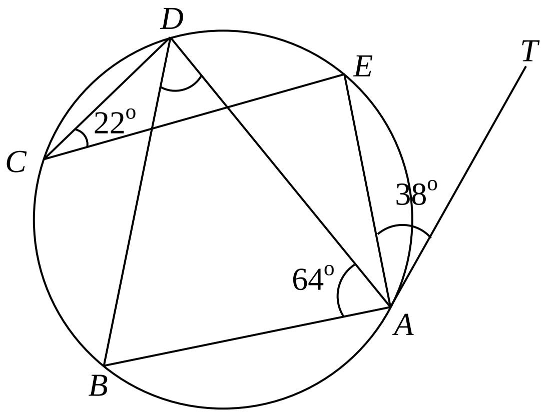
The area of the shaded region

= Area of sector BOC – Area of $\triangle COD$

$= \pi(12)^2 \times \dfrac{30^o}{360^o} - \dfrac{36\sin 45^o}{\sin 105^o}$

= 11 cm^2 (correct to the nearest cm^2)

39. B



$\angle DAE = \angle DCE$ ($\angle$s in the same segment)

$= 22^o$

$\angle DBA = \angle DAT$ ($\angle$ in alt. segment)

$= \angle DAE + \angle EAT$

$= 22^o + 38^o$

$= 60^o$

$\angle ADB + \angle DBA + \angle BAD = 180^o$ ($\angle$ sum of $\triangle$)

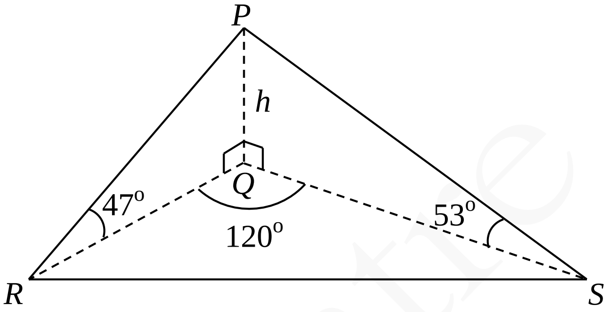
$\angle ADB + 60^o + 64^o = 180^o$

∴ $\angle ADB = 56^o$

40. C

Let $PQ = h$. Then, $PR = \dfrac{h}{\sin 47°}$, $PS = \dfrac{h}{\sin 53°}$, $QR = \dfrac{h}{\tan 47°}$, $QS = \dfrac{h}{\tan 53°}$.

Consider $\triangle QRS$, by cosine formula,



$RS^2 = QR^2 + QS^2 - 2(QR)(QS)\cos\angle RQS$

$$= \left(\frac{h}{\tan 47°}\right)^2 + \left(\frac{h}{\tan 53°}\right)^2 - 2\left(\frac{h}{\tan 47°}\right)\left(\frac{h}{\tan 53°}\right)\cos 120°$$

$$= \left(\frac{h}{\tan 47°}\right)^2 + \left(\frac{h}{\tan 53°}\right)^2 - 2\left(\frac{h}{\tan 47°}\right)\left(\frac{h}{\tan 53°}\right)(-0.5)$$

$$= \left(\frac{h}{\tan 47°}\right)^2 + \left(\frac{h}{\tan 53°}\right)^2 + \frac{h^2}{\tan 47° \tan 53°}$$

Consider $\triangle PRS$, by cosine formula,

$RS^2 = PR^2 + PS^2 - 2(PR)(PS)\cos\angle RPS$

$$= \left(\frac{h}{\sin 47°}\right)^2 + \left(\frac{h}{\sin 53°}\right)^2 - 2\left(\frac{h}{\sin 47°}\right)\left(\frac{h}{\sin 53°}\right)\cos\angle RPS$$

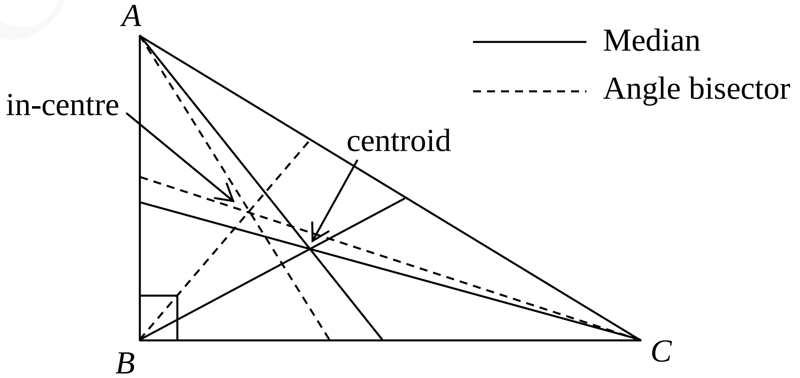
$$= \left(\frac{h}{\sin 47°}\right)^2 + \left(\frac{h}{\sin 53°}\right)^2 - \left(\frac{2h^2}{\sin 47° \sin 53°}\right)\cos\angle RPS$$

$$\therefore \quad \left(\frac{h}{\sin 47°}\right)^2 + \left(\frac{h}{\sin 53°}\right)^2 - \left(\frac{2h^2}{\sin 47° \sin 53°}\right)\cos\angle RPS = \left(\frac{h}{\tan 47°}\right)^2 + \left(\frac{h}{\tan 53°}\right)^2 + \frac{h^2}{\tan 47° \tan 53°}$$

$$\cos\angle RPS = \frac{\sin 47° \sin 53°}{2}\left(\frac{1}{\sin^2 47°} + \frac{1}{\sin^2 53°} - \frac{1}{\tan^2 47°} - \frac{1}{\tan^2 53°} - \frac{1}{\tan 47° \tan 53°}\right)$$

$\angle RPS = 68°$ (correct to the nearest degree)

41. B



B is the orthocentre.

∴ I is NOT true.

The centroid must lie inside $\triangle ABC$.

∴ II is true.

The in-centre must lie inside $\triangle ABC$.

∴ III is NOT true.

42. C

The required probability

= 1 – P("no blue cup is drawn")

$= 1 - \dfrac{C_6^{11}}{C_6^{19}}$ [choose from green cups and red cups]

$= \dfrac{635}{646}$

43. D

The required probability = 1 – P("Susan answers all questions correctly")

$$= 1 - \left(\frac{1}{3}\right)\left(\frac{1}{5}\right)\left(\frac{1}{7}\right)$$

$$= \frac{104}{105}$$

44. B

Let μ be the mean of the examination scores. Then,

$$\frac{69-\mu}{8} = 0.5$$

$\mu = 65$

∴ John's examination score

= 8(–1.5) + 65

= 53 marks

45. A

As each number of the set is multiplied by 6 and then 5 is added to each resulting number, the new mean is also multiplied by 6 and then 5 is added.

∴ I is true.

The new range is $6r$. Adding 5 to each number does not affect the range.

∴ II is NOT true.

The new variance is 6^2v i.e. $36v$. Adding 5 to each number does not affect the variance.

∴ III is NOT true.