# Suggested Solution for 2024 HKDSE Mathematics(core) Multiple Choice Questions

1. (

$$(x+3y)^{2} - (x-3y)^{2}$$

$$= [(x+3y) + (x-3y)][(x+3y) - (x-3y)]$$

$$= (2x)(6y)$$

$$= 12xy$$

2. D

$$\frac{(2\alpha)^3}{(4\alpha^{-5})^{-1}}$$

$$= \frac{2^3 \alpha^3}{4^{-1} \alpha^5}$$

$$= \frac{8 \cdot 4}{\alpha^{5-3}}$$

$$= \frac{32}{\alpha^2}$$

3. A

$$k = \frac{5}{2m} + n$$

$$\frac{5}{2m} = k - n$$

$$\frac{2m}{5} = \frac{1}{k - n}$$

$$m=\frac{5}{2(k-n)}$$

Alternatively

$$k = \frac{5}{2m} + n$$

$$2km = 5 + 2mn$$

$$2km - 2mn = 5$$

$$2m(k-n)=5$$

$$m=\frac{5}{2(k-n)}$$

4. A

$$\sqrt{333} \approx 18.24828759...$$

 $\approx$  18 (correct to the nearest integer)

 $\approx$  18.25 (correct to 2 decimal places)

 $\approx$  18.2 (correct to 3 significant figures)

≈ 18.2483 (correct to 4 decimal places)

### 5. B

Let x and y be the price of an apple and a lemon respectively.

$$2x + 3y = 38 \dots (1)$$

$$3x + 2y = 47 \dots (2)$$

Solving (1) and (2),

$$x = 13 \text{ and } y = 4$$

... The price of 4 apples and 7 lemons

$$= \$(4x + 7y)$$

$$=$$
\$[4(13) + 7(4)]

$$=$$
 \$80

$$4x^2 + 2ax + 3a \equiv x(4x+b) + 2c$$
$$\equiv 4x^2 + bx + 2c$$

$$\therefore$$
 2a = b and 3a = 2c

i.e. 
$$a:b=1:2=2:4$$
 and  $a:c=2:3$ 

$$\therefore$$
 *a* : *b* : *c* = 2 : 4 : 3

x = m - 1 or x = 4 - m

#### 7. B

$$x^{2} - 3x = (m-1)^{2} - 3(m-1)$$

$$x^{2} - (m-1)^{2} - 3x + 3(m-1) = 0$$

$$[x + (m-1)][x - (m-1)] - 3[x - (m-1)] = 0$$

$$[x - (m-1)][x + (m-1) - 3] = 0$$

$$[x - (m-1)][x + (m-4)] = 0$$

# Alternatively

$$x^{2} - 3x = (m-1)^{2} - 3(m-1)$$

$$x^{2} - 3x = (m-1)(m-1-3) = (m-1)(m-4)$$

$$x^{2} - 3x - (m-1)(m-4) = 0$$

By cross method,

$$[x - (m-1)][x + (m-4)] = 0$$

$$x = m - 1$$
 or  $x = 4 - m$ 

$$r$$
  $(m-4)$ 

By quadratic formula,

$$x = \frac{-(-3)\pm\sqrt{(-3)^2 - 4[-(m-1)(m-4)]}}{2}$$

$$= \frac{3\pm\sqrt{4m^2 - 20m + 25}}{2}$$

$$= \frac{3\pm\sqrt{(2m-5)^2}}{2}$$

$$= \frac{3+(2m-5)}{2} \text{ or } \frac{3-(2m-5)}{2}$$

$$= m-1 \text{ or } x = 4-m$$

$$g(1) = g(2)$$

$$\rightarrow$$
  $(1+1)(1+a) = (2+1)(2+a)$ 

$$\Rightarrow$$
  $a = -4$ 

$$g(-4) = (-4 + 1)[-4 + (-4)] = 24$$

By Factor theorem,

$$f(-k) = 0$$

i.e. 
$$(-k)^3 + k(-k)^2 + 5(-k) + 10 = 0$$

$$\rightarrow$$
  $k=2$ 

$$\therefore f(x) = x^3 + 2x^2 + 5x + 10$$

By Remainder theorem,

Remainder = 
$$f(-1)$$

$$= (-1)^3 + 2(-1)^2 + 5(-1) + 10$$

$$= 6$$

10. B

$$\frac{1-x}{2} \ge 4 \text{ or } 7 + 5x \le -3$$

$$1 - x \ge 8 \text{ or } 5x \le -10$$

$$x \le -7 \text{ or } x \le -2$$

$$\therefore x \leq -2$$

11. C

Let *x* be the number of students in the school.

$$x \times 40\% \times \beta\% + x(1 - 40\%) \times 30\% = x \times 40\%$$

$$\beta = 55$$

12. A

Average speed = 
$$\frac{60 \times 18 + 40 \times 27}{18 + 27}$$

$$=48 \text{ km/h}$$

13. C

Let  $z = \frac{kx^2}{y}$  where k is a constant. Let z' be the new value of z. Then,

$$z' = \frac{k[(1+20\%)x]^2}{(1-20\%)y}$$

$$=\frac{1.8kx^2}{v}$$

$$= 1.8z$$

% change in z

$$= \frac{1.8z - z}{z} \times 100\%$$

14. A

$$y = 2(6-x)^2 - 7$$
$$= 2(x-6)^2 - 7$$

$$\therefore a > 0$$

... The graph opens upwards.

Note that:

Vertex = (6, -7)  $\rightarrow$  The graph cuts the x-axis at two points. i.e. B is not true.

Substitute x = 0, y-intercept =  $2(0-6)^2 - 7 = 65 \neq -7$  i.e. C is not true.

$$2[6 - (-6)]^2 - 7 = 281 \neq -7$$
 i.e. D is not true.

# 15. D

Let r cm and  $\theta$  cm be the radius and the angle of the sector respectively.

$$\pi r^2 \times \frac{\theta}{360^o} = 80\pi \dots (1)$$

$$2\pi r \times \frac{\theta}{360^{\circ}} = 8\pi...(2)$$

Solving (1) and (2), r = 20 and  $\theta = 72^{\circ}$ 

#### 16. D

Let 32k and 15k be the heights of the right circular cylinder and the right circular cone respectively. Let r cm be the base radius of the circular cone.

$$\frac{\frac{1}{3}\pi r^2(15k)}{\pi(25)^2(32k)} = \frac{9}{10}$$

$$r = 60$$

#### 17. C

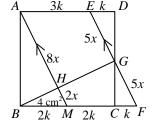
Note that AMFE is a parallelogram. Let ED = k. Then, AE = 3k, BM = MC = 2k and CF = k.

Note also that  $\triangle BHM \sim \triangle BGF$ .

$$\frac{HM}{GF} = \frac{BM}{BF} = \frac{2k}{2k+2k+k} = \frac{2}{5}$$

$$\frac{Area\ of\ \Delta BGF}{Area\ of\ \Delta BHM}\ =\ \left(\frac{GF}{HM}\right)^2 =\ \left(\frac{5}{2}\right)^2 =\ \frac{25}{4}$$

$$\therefore$$
 Area of  $\triangle BGF = 25 \text{ cm}^2$ 



Then, area of trapezium FGHM = area of  $\triangle BGF$  – area of  $\triangle BHM$  = 25 – 4 = 21 cm<sup>2</sup>

Note that  $\triangle DEG \cong \triangle CFG$  and G is the mid-point of EF.

Let 
$$EG = GF = 5x$$
. Then,  $HM = 2x$  and  $AH = 8x$ .

Let h cm be the common height of trapezium AEGH and FGHM. Then,

$$\frac{Area\ of\ trapezium\ AEGH}{Area\ of\ trapezium\ FGHM}\ =\ \frac{(5x+8x)h/2}{(2x+5x)h/2}\ =\ \frac{13}{7}$$

$$\therefore$$
 Area of trapezium  $AEGH = 21 \times \frac{13}{7} = 39 \text{ cm}^2$ 

$$BC^2 + BD^2 = 5^2 + 12^2 = 13^2 = CD^2$$

 $\therefore$   $\angle CBD$  is a right  $\angle$  (Converse of Pythagoras' Theorem)

By Pythagoras' Theorem,

$$AB^2 + BD^2 = AD^2$$

$$AB^2 + 12^2 = 37^2$$

$$AB = 35$$

i.e. 
$$AC = 5 + 35 = 40$$
 cm

By Pythagoras' Theorem,

$$AC^2 + CE^2 = AE^2$$

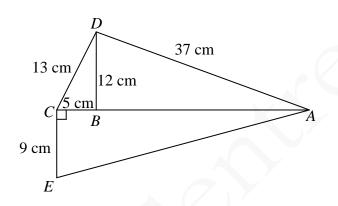
$$40^2 + 9^2 = AE^2$$

$$AE = 41$$

$$\therefore \text{ Perimeter of } ADCE = AD + DC + CE + EA$$

$$= 37 + 13 + 9 + 41$$

$$= 100 \text{ cm}$$



# 19. D

Draw parallel lines as shown in the right.

According to the diagram on the right,

$$a + p = 360^{\circ} (\angle s \text{ at a pt.})$$

$$a = 360^{\circ} - p$$

$$b = a = 360^{\circ} - p \text{ (alt. } \angle \text{ s, // lines)}$$

$$c = q - b = q - (360^{\circ} - p) = p + q - 360^{\circ}$$

$$d = c = p + q - 360^{\circ}$$
 (alt.  $\angle$ s, // lines)

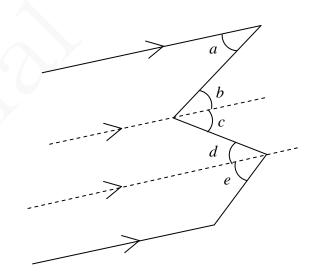
$$e + s = 180^{\circ}$$
 (int.  $\angle s$ , // lines)

$$e = 180^{\circ} - s$$

$$d + e + r = 360^{\circ}$$
 ( $\angle$ s at a pt.)

$$(p+q-360^{\circ})+(180^{\circ}-s)+r=360^{\circ}$$

i.e. 
$$p + q + r - s = 540^{\circ}$$



#### 20. D

Let n be the number of sides of the polygon.

$$180^{\circ} \times (n-2) = 900^{\circ}$$

$$n = 7$$

The number of diagonals =  $\frac{7 \times 6}{2}$  = 21

.. I is NOT true.

II and III are true for a regular heptagon.

### 21. B

Let  $\angle FCH = x$  and  $\angle GCH = y$ . Note that  $x + y = 90^{\circ}$ .

Then, 
$$\angle IFC = \angle FCH = x$$
 (alt.  $\angle s$ ,  $BH // EF$ )

$$\angle BCE = \angle GCH = y \text{ (vert. opp. } \angle s)$$

$$\angle DCE = \angle BCE = y$$
 (properties of rhombus)

$$\angle FCD + \angle DCE = 90^{\circ}$$

$$\rightarrow$$
  $\angle FCD = 90^{\circ} - y = x = \angle IFC$ 

$$\therefore$$
 CI = FI (sides opp. equal  $\angle$ s)

.. I is true.

 $AC \perp DB$  (properties of rhombus)

i.e. 
$$\angle CEB = 90^{\circ}$$

$$\angle CBE + \angle CEB + \angle BCE = 180^{\circ} (\angle \text{ sum of } \Delta)$$

$$\angle CBE + 90^{\circ} + y = 180^{\circ}$$

$$\rightarrow$$
  $\angle CBE = 90^{\circ} - y = x$ 

$$\angle ABE = \angle CBE = x$$
 (properties of rhombus)

but 
$$\angle GCH = y$$

:. II may not be true.

Note that owing to the properties of rhombus,  $\triangle ADE$ ,  $\triangle ABE$ ,  $\triangle CDE$  and  $\triangle CBE$  are congruent.

Note that  $\triangle CBE$  and  $\triangle EFC$  are congruent (ASA).

Note that  $\triangle EFC$  and  $\triangle HCF$  are congruent (ASA).

- $\triangle ADE \cong \triangle HCF$
- .. III is true.

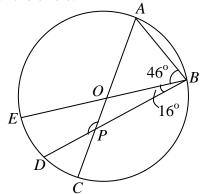
### 22. B

Mark the intersection of AC and BE as O which is the centre of the circle.

$$\angle OAB = \angle OBA = 46^{\circ}$$
 (base  $\angle$  s, isos.  $\triangle$ )

$$\angle POB = \angle OBA + \angle OAB \text{ (ext. } \angle \text{ of } \Delta\text{)}$$
  
=  $46^{\circ} + 46^{\circ}$   
=  $92^{\circ}$ 

$$\angle APD = \angle POB + \angle DBE \text{ (ext. } \angle \text{ of } \Delta\text{)}$$
  
=  $92^{\circ} + 16^{\circ}$   
=  $108^{\circ}$ 



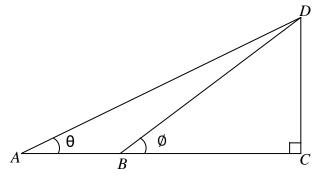
# 23. A

$$\frac{DC}{AD} = \sin \theta \dots (1)$$

$$\frac{DC}{RC} = \tan \emptyset \dots (2)$$

Combining (1) and (2),

$$\frac{BC}{AD} = \frac{\sin \theta}{\tan \theta}$$



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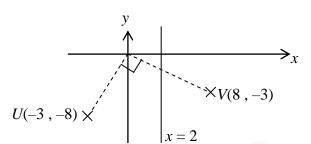
### 24. A

After clockwise rotation, the coordinates of V are (8, -3).

Let the x-coordinate of W be x'.

$$2 - x' = 8 - 2$$

→ 
$$x' = -4$$



### 25. C

Let P = (x, y). Then,

$$x - y + 13 = 0 \dots (1)$$
 [: P lies on the straight line]

$$\therefore AP = PB$$

$$\therefore [x-(-3)]^2 + (y-1)^2 = [x-(-7)]^2 + [y-(-5)]^2$$

$$\Rightarrow$$
 2x + 3y + 16 = 0 ... (2)

Solving (1) and (2), we have x = -11 and y = 2

Rewrite the equations of the straight lines in slope-intercept form.

$$\begin{cases} y = \frac{3}{4}x - \frac{7k}{8} \\ y = -\frac{k}{12}x + \frac{5}{12} \end{cases}$$

... If the two lines are parallel and the *y*-intercepts of them are not equal, then they do not intersect with each other.

i.e. 
$$-\frac{k}{12} = \frac{3}{4}$$

$$\rightarrow$$
  $k = -9$ 

Check that the y-intercepts of the straight lines are  $-\frac{7(-9)}{8}$  i.e.  $\frac{63}{8}$  and  $\frac{5}{12}$  which are not equal.

# 27. D

Note that  $C: x^2 + y^2 - 2x + 4y - \frac{4}{3} = 0$ . Let G be the centre of the circle. Then,

$$G = (\frac{-(-2)}{2}, \frac{-4}{2}) = (1, -2)$$

Radius, 
$$r = \sqrt{(1)^2 + (-2)^2 - \left(-\frac{4}{3}\right)} = \sqrt{\frac{19}{3}}$$

$$OG = \sqrt{(1-0)^2 + (-2-0)^2} = \sqrt{5} < \sqrt{\frac{19}{3}}$$

- $\therefore$  O lies inside C.
- I is true.

The circumference of  $C = 2\pi \sqrt{\frac{19}{3}} < 16$ 

- II is true.
- The y-coordinate of G = -2
- III is true.

# 28. C

29. B

Refer to the table on the right. The numbers circled are not less than 12.

The required probability

$$=\frac{7}{15}$$

	1	2	3	4	5	6
1	X	2	3	4	5	6
2	X	X	6	8	10	(12)
3	X	X	X	(12)	(15)	)8
4	X	X	X	X	(20)	24
5	X	X	X	X	X	(30)

1st card

2<sup>nd</sup> card

From the box-and-whisker diagram, range = 472 - 136 = 336

Inter-quartile range = m - 163

- $\therefore$  3(m-163) 336
- $\rightarrow m = 275$

### 30. D

$$\frac{5+5+5+6+9+9+11+13+m+n}{10} = 7$$

$$\therefore$$
  $m+n=7$ 

Since m and n cannot be 6, 9, 11 or 13 at the same time, the mode must be 5.

: II is true.

Possible values of m and n and the corresponding values of the median and standard deviation are shown below:

<u>m</u>	<u>n</u>	<u>median</u>	standard deviation
1	6	6	3.31662479
2	5	5.5	3.193743885
3	4	5.5	3.130495168

... Both II and III are true.

# 31. B

Taking the smallest degree from u, v and w, the H.C.F. is  $u^2vw$ .

# 32. A

$$\begin{split} &AF000000000BC_{16}\\ &= 10\times 16^{12} + 15\times 16^{11} + 11\times 16^{1} + 12\times 16^{0}\\ &= (10\times 16 + 15)\times 16^{11} + 188\\ &= 175\times 16^{11} + 188 \end{split}$$

$$\begin{cases} x = \log_2 y - 2 \dots (1) \\ (\log_2 y)^2 = 5\log_2 y + x - 7 \dots (2) \\ \text{Substitute (1) into (2),} \\ (\log_2 y)^2 = 5\log_2 y + (\log_2 y - 2) - 7 \\ (\log_2 y)^2 - 6\log_2 y + 9 = 0 \\ (\log_2 y - 3)^2 = 0 \\ \log_2 y = 3 \\ y = 8 \end{cases}$$

# 34. D

Slope of the graph = -16

Using slope-intercept form of a straight line equation (i.e. y = mx + c), we get

$$y^3 = -16\sqrt{x} + 32$$

When 
$$x = 36$$
,

$$y^3 = -16\sqrt{36} + 32$$
  
= -64

$$y = -4$$

### 35. A

$$z = (a-5)i + \frac{(a+2)i}{2+i}$$

$$= (a-5)i + \frac{(a+2)i}{2+i} \times \frac{2-i}{2-i}$$

$$= (a-5)i + \frac{2ai+4i-ai^2-2i^2}{2^2+1}$$

$$= (a-5)i + \frac{a+2+(2a+4)i}{5}$$

$$= \frac{a+2}{5} + \frac{5(a-5)+2a+4}{5}i$$

$$= \frac{a+2}{5} + \frac{7a-21}{5}i$$

 $\therefore$  z is a real number.

$$\therefore \frac{7a-21}{5}=0$$

$$a = 3$$

Then, 
$$z = \frac{3+2}{5} = 1$$

$$a - z = 2$$

### 36. C

Let T(n) be the *n*th term of the sequence. Then,

$$T(n) = S(n) - S(n-1)$$

$$= n(2n+3) - (n-1)[2(n-1)+3]$$

$$= 4n+1$$

: II is true.

Note that T(n) - T(n-1) = 4n + 1 - [4(n-1) + 1] = 4

... The sequence is an arithmetic sequence.

:. III is true.

$$4n + 1 = 14$$

$$n = \frac{13}{4}$$
 is not an integer.

:. I is NOT true.

$$\begin{cases} x - 2y = 1 \dots (1) \\ x + 4y = 13 \dots (2) \\ 2x - y = -1 \dots (3) \end{cases}$$

Solving (1) and (2), the intersection of (1) and (2) is (5, 2).

Similarly, the intersection of (1) and (3) is (-1, -1)

while that of (2) and (3) is (1, 3).

Let 
$$P(x, y) = 5x - 2y + c$$

$$P(5, 2) = 5(5) - 2(2) + c \ge 22$$

$$\rightarrow$$
  $c \ge 1$ 

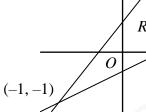
$$P(-1,-1) = 5(-1) - 2(-1) + c \ge 22$$
  $c \ge 25$ 

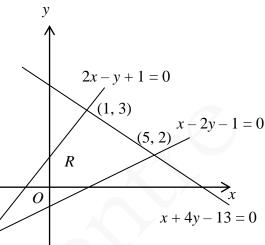
$$\rightarrow c > 25$$

$$P(1,3) = 5(1) - 2(3) + c \ge 22$$

$$\rightarrow$$
  $c \ge 23$ 

 $\therefore$  Combining the results,  $c \ge 25$ .





38. B

Join CD.

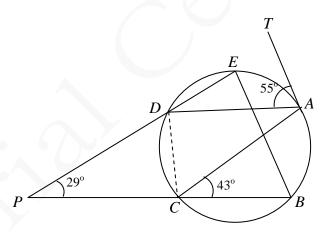
$$\angle ACD = \angle DAT = 55^{\circ} (\angle \text{ in alt. segment})$$

$$\angle CDP + \angle CPD = \angle DCB \text{ (ext.} \angle \text{ of } \Delta)$$

$$\angle CDP + 29^{\circ} = 55^{\circ} + 43^{\circ}$$

$$\angle CDP = 69^{\circ}$$

$$\angle CBE = \angle CDP = 69^{\circ} \text{ (ext.} \angle = \text{int. opp.} \angle)$$



39. A

$$4\cos^2\theta - 3\cos\theta - 1 = 0$$

$$(4\cos\theta + 1)(\cos\theta - 1) = 0$$

$$\cos \theta = -\frac{1}{4} \text{ or } 1$$

 $\theta = 105^{\circ}$ , 256° or 360° for  $0^{\circ} < \theta \le 360^{\circ}$ .

Let the length of SQ be h. Then,

$$\frac{SQ}{RQ} = \tan \angle QRS$$

$$\frac{h}{RQ} = \tan 45^{\circ}$$
 i.e.  $RQ = h$ 

$$\frac{SQ}{PO} = \tan \angle QPS$$

$$\frac{h}{PQ} = \tan 30^{\circ}$$
 i.e.  $PQ = \sqrt{3}h$ 

By Pythagoras's Theorem,

$$PR^{2} = PQ^{2} + RQ^{2}$$
$$= (\sqrt{3}h)^{2} + (h)^{2}$$
$$= 4h^{2}$$

$$\frac{SQ}{PS} = \sin \angle QPS$$

$$\frac{h}{PS} = \sin 30^{\circ}$$
 i.e.  $PS = 2h$ 

$$\frac{SQ}{RS} = \sin \angle QRS$$

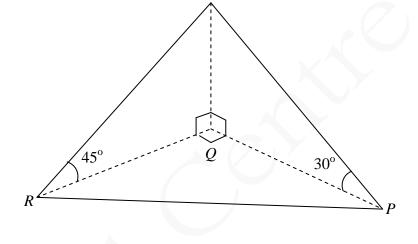
$$\frac{h}{RS} = \sin 45^{\circ}$$
 i.e.  $RS = \sqrt{2}h$ 

By cosine formula,

$$PS^2 = PR^2 + RS^2 - 2(PR)(RS)\cos \angle PRS$$

$$(2h)^2 = 4h^2 + (\sqrt{2}h)^2 - 2(2h)(\sqrt{2}h)\cos \angle PRS$$

$$\cos \angle PRS = \frac{\sqrt{2}}{4}$$



# 41. A

$$\angle QPR = 136^{\circ}$$

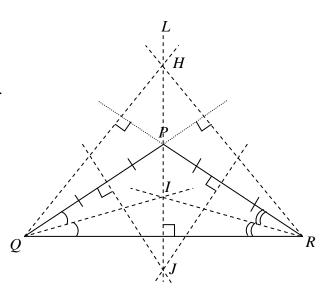
Refer to the diagram on the right.

Note that G, H, I and J lies on the same straight line L.

I is true.

II is true.

III is NOT true.



Referring to the diagram on the right, the managers can take any two positions indicated by the arrows.

Number of different queues

$$= P_2^8 \times P_7^7$$
= 282 240

# Alternatively

Suppose the two managers are next to each other. Take them as one person. It will be a queue of 8 persons. In addition, the 2 managers can interchange their positions. Therefore, the number of queues the two mangers are next to each other =  $P_8^8 \times P_2^2$ 

Hence, the number of different queues the managers are not next to each other

$$= P_9^9 - P_8^8 \times P_2^2$$
$$= 282 \ 240$$

### 43. D

The required probability

#### 44. D

Rearrange the scores in ascending order.

10 13 16 17 19 25 26 28 30 30 32 39 
$$Median = \frac{25+26}{2} = 25.5$$

.. I is NOT true.

Mean, 
$$\bar{x} = \frac{10+13+16+17+19+25+26+28+30+30+32+39}{12} = 23.75$$

Standard deviation,  $\sigma \approx 8.347903929$ 

.. III is true.

The standard score of the largest datum(39)  $\approx \frac{39-23.75}{8.347903929} \approx 1.826805882 < 2$ 

.. II is true.

### 45. B

New variance = 
$$9^2 \times 16$$
  
New standard deviation =  $\sqrt{9^2 \times 16}$   
= 36